JAN 29 1920
UNIV. OF MICHIGAN

Annual Subscription, Seven Dollars
New rate, effective January 1, 1920

VOLUME XIV SECOND SERIES NUMBER 6

THE PHYSICAL REVIEW

A JOURNAL OF EXPERIMENTAL AND THEORETICAL PHYSICS

CONDUCTED BY

THE AMERICAN PHYSICAL SOCIETY

BOARD OF EDITORS

FREDERICK BEDELL, Managing Editor

N. E. DORSEY G. K. BURGESS HENRY CREW
WM. DUANE A. D. COLE L. V. KING
O. M. STEWART A. C. LUNN H. S. UHLER

DECEMBER, 1919

The Optical Constants of Liquid Alloys. C. V. KENT 459

On the Maintenance of Vibrations of Wires by Electrical Heating. N. C. KRISHNAIYAR 490

On the Diffraction Theory of Microscopic Vision. PHANINDRA NATH GHOSH 497

Resonance and Ionization Potentials for Electrons in the Monatomic Gases Argon, Neon and Helium. H. C. RENTSCHLER 503

On the X-Ray Absorption Frequencies Characteristic of the Chemical Elements. WILLIAM DUANE AND KANG-FUH-HU 518

On the X-Ray Absorption Frequencies Characteristic of the Chemical Elements. WILLIAM DUANE AND TAKEO SHIMIZU 522

The Relation between the Intensity of General X-Radiation and the Atomic Number of the Anticathode. WILLIAM DUANE AND TAKEO SHIMIZU 525

Proceedings of the American Physical Society

(Contents continued on back cover page)

PUBLISHED BY
THE AMERICAN PHYSICAL SOCIETY
LANCASTER, PA., AND ITHACA, N. Y.

Entered at the post-office, at Lancaster, Pa., as second class matter

THE AMERICAN PHYSICAL SOCIETY

OFFICERS OF THE SOCIETY

President:	J. S. AMES	Johns Hopkins University, Baltimore, Md.
Vice President:	THEODORE LYMAN	Harvard University, Cambridge, Mass.
Secretary:	D. C. MILLER	Case School of Applied Science, Cleveland, Ohio
Treasurer:	G. B. PEGRAM	Columbia University, New York City
Editor:	F. BEDELL	Cornell University, Ithaca, N. Y.
Local Secretary for the Pacific Coast:	E. P. LEWIS	University of California, Berkeley, Cal.

The Council of the Society consists of the president, vice-president, secretary, treasurer, managing editor, all living Past Presidents and eight elected members as follows:

Past Presidents: A. A. Michelson, A. G. Webster, Carl Barus, E. L. Nichols, Henry Crew, W. F. Magie, Ernest Merritt, R. A. Millikan, H. A. Bumstead.

Elected Members: Irving Langmuir, Max Mason, G. O. Squier, H. A. Wilson, A. L. Day, G. F. Hull, G. K. Burgess, J. C. McLennan.

A complete list of past officers is given on a following page.

THE PHYSICAL REVIEW

Annual Subscription, throughout the Postal Union, $7.00

Single Copies, postpaid, $.75

THE PHYSICAL REVIEW is conducted by a BOARD OF EDITORS, named on the outside cover.

Subscriptions should be sent to THE PHYSICAL REVIEW, *41 North Queen Street, Lancaster Pa.*, or *Ithaca, N. Y.*

Two volumes of THE PHYSICAL REVIEW are published annually, these volumes beginning in January and July, respectively, and containing six numbers each.

A limited number of complete sets of the REVIEW, both First and Second Series, will be supplied at the price of $3.50 per volumes for all volumes, carriage extra. When they can be furnished without breaking a set, separate back volumes will be supplied at the same price, and single back numbers will be mailed to any address in the Postal Union at the price of $.75 per copy.

Back numbers will be furnished to members at a discount of ten per cent., provided a statement of membership accompanies the order. When a member's name has been dropped from the mailing list of the Review on account of non-payment of dues, his name will be restored to the mailing list when dues are paid; and any numbers that he has failed to receive in the interim will be furnished without cost upon application to the Review. *All complaints for failure to receive the Review should be addressed to the Review and not to any officer of the Society.*

First Series. From July 1, 1893, to January 1, 1913, the REVIEW was conducted by Edward L. Nichols, Ernest Merritt and Frederick Bedell, with the financial support of Cornell University. During this period thirty-five volumes were issued, constituting the First Series. The first four volumes of the First Series were issued annually; subsequent volumes were issued semi-annually.

Second Series. On January first, 1913, the Review was transferred to the American Physical Society. A second series was then begun, the January, 1913, number being designated Volume I., Number 1, Second Series. (An announcement of this transfer will be found on the first page of the new series.)

For advertising rates, address THE PHYSICAL REVIEW, *Ithaca, N. Y.* Change of copy in standing advertisements should be sent direct to the printers, THE NEW ERA PRINTING CO., *Lancaster, Pa.*

Contributors to the REVIEW are referred to page two of the advertisements for further information.

Permission is given to republish any article appearing in the REVIEW, provided proper credit is given.

Address Correspondence to

THE PHYSICAL REVIEW, ITHACA, N. Y.

Second Series. *December, 1919.* *Vol. XIV., No. 6*

THE

PHYSICAL REVIEW.

THE OPTICAL CONSTANTS OF LIQUID ALLOYS.

By C. V. Kent.

Synopsis.

This paper describes a determination of the two optical constants of molten metals, the index of refraction, ν, and the index of absorption, κ, from the reflection of polarized light at molten-metal surfaces, placed in an inactive atmosphere.

The *purpose* of this experiment is to throw some light upon the behaviour of free electrons in metals.

Method.—The constants, ν and κ, are computed from the phase change Δ, and the azimuth change ψ, of polarized light reflected from the surface of the liquid metal at an angle of incidence ϕ. A Jamin circle is employed, the azimuth of the analyzer being determined by a bi-field of new design.

A furnace for melting the metals is made up by winding a few yards of nichrome wire on a hollow iron cone, thus maintaining a temperature of about 400° C. The whole furnace is operated in a water-cooled container through which a stream of hydrogen is made to flow continuously.

The metals employed are bismuth, cadmium, tin, lead and binary alloys of these. As a source of light, three strong rays from a quartz mercury arc are used.

Results.—A good optical surface is obtained, free from the scratches and impurities of polishing materials.

Values of $\nu^2 - \kappa^2$ and $2\nu\kappa$, are plotted as functions of the composition (atomic concentration) of each alloy. The parameters of the free electrons are also computed and plotted as functions of the composition of the alloy. These parameters are: (1) the product Ne^2/m, where the symbols have the meanings usually given in the electron theory, and (2) the frequency of impact of a free electron with the molecules of the metal.

From these optically determined free electron parameters, the electrical resistivities of the pure liquid metals and alloys are computed. In every case the agreement with the known electrical resistivity is within the limits of experimental error. A possible explanation of the disagreement between the values of resistivities for solid metals as computed from optical data and from direct electrical measurement, is suggested.

The results seem to indicate that the simple free electron theory of Drude and others is applicable to molten metals.

I. INTRODUCTION.

THE optical constants of metals (*i.e.*, the indices of refraction and absorption) are of some importance in the free electron theory. They afford a fairly direct method of obtaining a knowledge of the approximate magnitudes of some of the free electron parameters. However, the values of these optical constants as obtained experimentally for metals in the solid state, are unexpectedly unsatisfactory and inconsistent.

This is due to several causes. It is obvious that the necessary experimental data are most readily obtained by determining the changes produced in light when it is reflected from the metal to be examined; these changes being of intensity and phase. The accuracy of results is thus limited by the photometrical sensibility of the eye.

Furthermore, it is almost impossible to obtain plane surfaces which are *optically* pure, by ordinary grinding and polishing. Scratches and traces of abrasive agents, imperceptible to the naked eye, produce surfaces quite different from those of the pure metal. Minor,[1] Erochin,[2] Duncan[3] and others have minimized this source of error by using mirrors deposited on glass and examining the glass-metal inter-face. Their data show, however, that the optical constants vary with the method of producing the surfaces, whether by chemical processes, cathode deposition or casting. For these reasons, results obtained by different observers with the same metal, or even by the same observer with successive surfaces vary from 5 to 20 per cent.

The theoretical interpretation and correlation of data for different metals in the solid state is also open to objection. For example, the parameters controlling electron motion within the space lattice of a metallic crystal will be functions of direction of motion. It seems reasonable, therefore, to expect different values of the optical constants for different orientations. Thus, the optical constants of an extended solid metal surface are perhaps merely averages for the various crystals exposed.

In the course of an optical investigation of alloys, I decided to eliminate the above-named difficulties as far as possible, by restricting the work to liquid surfaces in an inactive atmosphere. By so doing, I could obtain surfaces which were perfectly smooth and practically free from contamination. Any small impurities would be either dissolved in the metal and be negligible in their effect, or, floating as solid aggregates, could be removed by mechanical means.

[1] R. S. Minor, Ann. d. Phys., 10, 581, 1893.
[2] P. Erochin, Ann. d. Phys., 39, 213, 1912.
[3] R. W. and R. C. Duncan, PHYS. REV., 1, 294, 1913.

The investigation was further restricted to pure metals and bi-metallic alloys in which there were no compounds. In this simple case, the alloy may be considered from a theoretical point of view to consist of three mutually dissolved components: (1) The molecules of the first component metal and (2) of the second, each preserving the characteristics peculiar to the molecules of the pure metals, and (3) that portion to which metallic conduction is due. This portion is the free electrons common to all metals, and contributed in the case of the alloys here tested, by the two component metals.

II. Previous Work.

A few determinations of the optical constants of alloys have been made, most of them for the various kinds of speculum metal. Bernouilli[1] has worked with the binary mixtures Ag-Te, Ag-Sn, Cd-Hg, Cu-Sn, Cu-Zn and Cu-Ni; in each case, only a small portion of the latter component was present. Littleton[2] investigated the effect of the carbon content in various steels, and in another paper has taken up the systematic study of iron-nickel, nickel-silicon, iron-manganese, aluminum-copper, copper-nickel and iron-copper alloys. His efforts were chiefly directed to tracing the effects of the presence of compounds of the components present. Recently L. K. Oppittz[3] has studied the binary alloys of silver with copper and platinum.

Practically no results have been obtained for liquid metals with the exception of mercury. Drude mentions his apparently unsuccessful attempts to work with melted tin; he also gives some values of the optical indices for mercury alloyed with 2 per cent. tin, and for liquid Wood's metal. No other data for liquid metals has been found.

III. Method.

This is the familiar method employed in the investigation of opaque substances. Light incident upon the reflecting surface to be examined, is plane polarized in a plane at an angle of 45 degrees to the plane of incidence. The difference of phase between the two components of the reflected light is determined by a phase compensator which again plane polarizes the light passing through it. The azimuth of the plane of polarization of this light is determined by an analyzer.

Let the angle of incidence be ϕ, the phase difference introduced by reflection be Δ, and the azimuth of the plane of polarization of the reflected light with respect to the plane of incidence be ψ. In conformity

[1] A. Bernouilli, Zeitschr. f. Elektrochemie, 15, 647, 1909.

[2] J. T. Littleton, Phys. Rev., 28, 306, 1912; 33, 453, 1912.

[3] L. K. Oppittz, Phys. Rev., 10, 156, 1917.

with the terminology of many investigators in this subject, I have adopted the optical constants ν and κ, which are equal to Drude's n and nk respectively. The following convenient reduction formulæ may then be used to calculate ν and κ.

$$\cos 2P = \sin 2\psi \cos \Delta, \qquad \tan Q = \tan 2\psi \sin \Delta,$$

$$S = \sin \phi \tan \phi \tan P,$$

$$\nu^2 - \kappa^2 = S^2 \cos 2Q + \sin^2 \phi = A,$$

$$2\nu\kappa = S^2 \sin 2Q = B.$$

Hence

$$2\nu^2 = \sqrt{A^2 + B^2} + A,$$

$$2\kappa^2 = \sqrt{A^2 + B^2} - A,$$

ν, κ, and the reflecting power at perpendicular incidence may then be found.

IV. Description of Apparatus.

Monochromatic Illuminator.—A Cooper-Hewitt quartz tube mercury arc was used as a source of illumination. Its light was separated into its component colors by a Wadsworth prism-mirror train. The three intense lines, 404 $\mu\mu$, 546 $\mu\mu$ and 578 $\mu\mu$ (violet, green and yellow) were used. The colored slit image of the slit in the Wadsworth train covered the collimator slit, the collimator lens being completely filled at all times with the light employed.

Spectroscope and Polarization Train.—The spectroscope was a "Cercle de Jamin" from the Société Genevoise and is evidently intended for this sort of work. As liquid surfaces were used the circle was turned into a vertical plane. The verniers of the telescope and collimator gave their position on the spectroscope circle to 30″.

The polarizer and analyzer were mounted on the objective lens end of the collimator and telescope respectively. The nicols were of the Glan-Thompson type with about 1 cm. aperture. They were perfect in action. Their divided circles could be ready by verniers to 6′.

The compensator was mounted between the polarizer and the reflecting surface. It was of the Soleil-Babinet type; the micrometer head of the screw moving the quartz wedge was divided into 100 parts, each corresponding roughly to about 1/2 degree. Thus by estimating to tenths of a division, an accuracy of about 3′ could be obtained.

The writer's experience with this compensator is identical with that of R. W. and R. G. Duncan, in that it is impossible to obtain uniform darkness over the whole field of view. With the compensator and analy-

zer adjusted for minimum illumination, if the telescope eyepiece was removed, the field of view presented a blotched or striated appearance. With the eyepiece in place, the collimator slit image was traversed by a dark and fairly narrow diagonal band, which shifted position very rapidly with slight changes in the compensator setting. It is probable that this defect in the action of the compensator is due to some irregularity of crystal structure in the quartz plates.

In order to use the compensator for its purpose the practice was adopted of taking all compensator readings when the center of the black band lay under the intersection of the telescope crosshairs. This was done both in the preliminary calibration of the compensator, *i.e.*, in the determination of the phase shift per division on the compensator micrometer head for each color of light used, and in the regular experimental work. In this latter work, what was measured was the phase shift necessary to bring the diagonal band back under the cross hairs when it was shifted by reflection. From the results of a number of preliminary experiments the writer is convinced that no errors were introduced by this method of using the compensator.

Bi-field.—Total extinction of the light after it was reflected could never be produced in this experimental work by any adjustments of compensator and analyzer. This was due to the following causes: the imperfection of the compensator described above; films of moisture or metal on the observation windows of the housing of the heater in which the metal was melted; sources of scattered light such as dust particles, etc., in the optical train between the nicol prisms. An azimuth bi-field was therefore absolutely necessary for determining the azimuth position of the analyzer.

An entirely new form of bi-field was invented by the writer for this purpose in 1913. It was subsequently described by Dr. F. E. Wright,[1] of the Geophysical Laboratory. As the mounting and use by the writer is quite different from the adaptation of Dr. Wright, a brief description will be given.

The bi-field (see Fig. 1) consists of two slips of glass mounted on, and rotating with the analyzer. They are situated, of course, in the beam of light from the polarizer to the analyzer. Satisfactory slips are easily cut from a good strain-free microscope slide. The slips abut along a common edge and are inclined to each other at a somewhat obtuse angle, perhaps 120 degrees. Their common edge is in (or perpendicular to) the plane of polarization of the analyzer and their faces make equal angles with the direction of the beam of light passing through them and the

[1] F. E. Wright, Wash. J. Sci., 4, 309, 1914.

analyzer. The common edge is also inclined at an angle with the direction of the beam of light; the inclination is variable in order to change the sensibility of the bi-field.

When the bi-field is in use, a short focus telescope is substituted for the eyepiece of the spectroscope telescope and focused upon the common edge of the glass slips. The field of view is thus divided in halves by this edge. The "crossed" or extinction position of the analyzer is denoted by equality of brightness of the halves of the field. The arrangement acts exactly as the usual quartz bi-field of variable sensitivity.

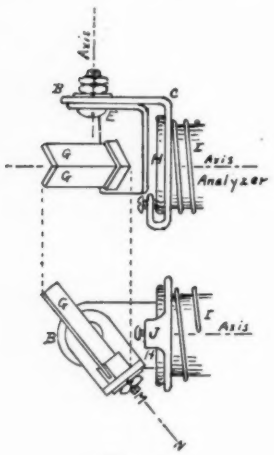


Fig. 1.

The dividing line between the halves of the field is practically eliminated (1) by mounting the glass slips with the exterior obtuse angle toward the incoming beam of light, and (2) by grinding the adjacent edges of the slips or plates to a rough fit. The interior edge of one plate is finished very sharply and overlaps the other. Thus this finely finished edge serves as a common dividing line and is practically invisible.

Multiply reflected light is present in the field of view except in a narrow band on each side of the dividing line. As the sensitivity is much reduced by this multiply reflected light, only the narrow bands are observed when obtaining the position of equal brightness.

In order to properly adjust the bi-field to the position described above it is mounted as shown in the accompanying figure. (Fig. 1).

The theory of its action is simple. Consider plane-polarized light incident upon one of the glass plates and resolve it into its two components in, and perpendicular to, the plane of incidence. By transmission these are unequally diminished with no phase difference introduced, according to the ordinary Fresnel laws. After transmission these again combine to form light of slightly diminished intensity, and on account of the inequality of diminution, plane-polarized in a slightly different plane from that of the incident light. In other words the light has been slightly diminished in intensity and *rotated through a small angle.* The light passing through the other plate will be likewise decreased in intensity and its plane of polarization also rotated, but in the opposite direction. If, now, the analyzer is in the "crossed" or total extinction position, the two glass plates are symmetrically situated, both with respect to the analyzer and the plane of polarization of the incident light.

In this case the lights passing through the two plates are equal in brightness and their planes of polarization equally rotated, but in opposite directions, with respect to the plane of polarization of the analyzing nicol. Therefore when they are observed through the analyzer the two halves of the field will be of equal brightness. When the analyzer (and glass plates) are in any other position, equality of brightness will not obtain.

The action of this bi-field is the same when it is mounted on the polarizer instead of the analyzer, and in the same position with respect to the plane of polarization of this nicol. The common edge may also be in a plane perpendicular to the plane of polarization of the nicol on which it is fastened. Owing to the mounting used, all adjustments of the bi-field for its proper action can be made optically by simple methods which need not be described here. The angle between the planes of polarization of the beams of light passing through the two plates, can be varied from 0 to about 12 degrees.

The arrangement described seems to have all the advantages of the quartz bi-field of variable sensibility. In addition its cost is insignificant and it is very easily constructed.

V. Furnace and Container.

These were mounted on a heavy laboratory tripod beside the spectroscope circle. They could be easily swung out from between the collimator and telescope at any time for adjustment, etc.

The necessary details of the heater and container for the hydrogen atmosphere above the liquid metal may be seen from the accompanying diagram (Fig. 2).

The heater, Q, consisted of about 8 m. of no. 28 nichrome wire imbedded in an asbestos water-glass cement. The liquid metal was held in the iron cone Q, whose surface was rubbed with steatite to prevent adherence. S is a protecting asbestos rim; an overhanging roof protects the binding posts R, from spilled metal. Wires from the heating coil are led outside the container through the tube D, in the base. The asbestos bottom of the heater is cut away over the exit tube E to allow for egress of detritus and samples of the metal used.

The container was made in two parts, both water-cooled. When in use these were cemented together by soft wax. The base of the container consisted of a sheet metal cylinder 12 cm. in diameter and 2 cm. high. The tubes A and B in this, are for the ingress and egress of water, D for the heater wires, and E for the dumping of detritus and samples into a beaker of water below, into which it projected. Hydrogen, which flowed slowly through the apparatus during observations, bubbled out through this tube.

The upper portion of the container was made of two concentric sheet iron cylinders soldered together at the bottom edge. The space between these was utilized for cooling water, the outer cylinder being open at the top. *F* and *G* are two short tubes to which the observation windows (two strain-free microscope slides) were fastened with soft wax. Light to and from the reflecting liquid metal surface passed through these plates practically perpendicularly. Deviations of a degree or so from this perpendicularity were found to produce no change in the analyzer settings.

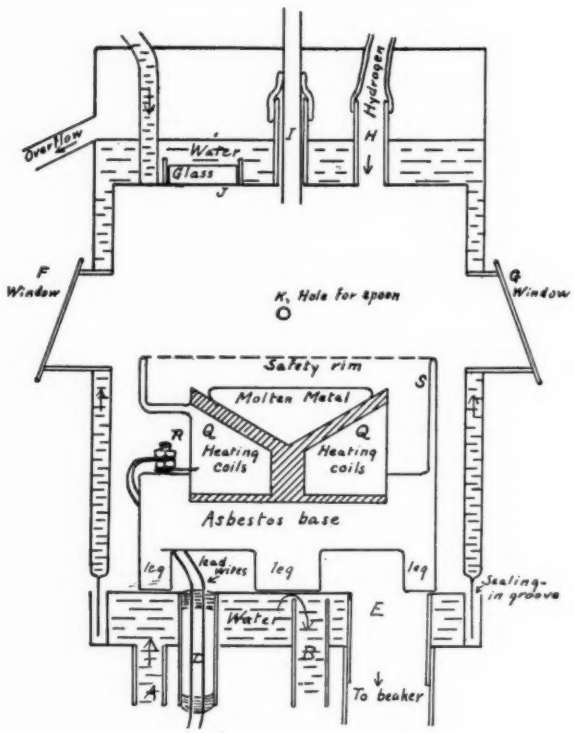


Fig. 2.

Tube *H* at the top of the container, was connected with a Kipp hydrogen generator. A glass tube slipped through tube *I*, the sliding joint being rendered gas tight by a gum rubber collar. This tube was used for the introduction of new metal when changing the composition of the alloy under observation. For this purpose it was pushed down until the lower end was immersed beneath the surface of the liquid metal. The component metal to be added was poured down through it in granular form, the tube corked and lifted up to its former position. Thus the interior of the container was at no time open to the air. *J* is an additional

glass window, for the general illumination of the interior of the container, if it should be needed.

At *K*, the sides of the inner and outer container walls were pressed together and soldered, and a small hole bored through both. A short brass collar was soldered over the hole on the outside. Over this was fastened a rubber finger stall. The handle of the "skimming spoon" (to be described later) was passed through this hole into the finger stall and could be manipulated from the outside.

It was impossible to obtain a clear liquid surface when the metal was first introduced into the heater and melted. The surface was therefore freed from oxide and other coarse impurities originally present by skimming with a specially formed spoon, whose handle projected into the finger stall on the outside, as previously described. The bowl of the spoon was a sheet iron cone of practically the same shape and dimensions as the cone in which the metal was melted. A hole 1 mm. in diameter was bored at the bottom point of this cone. The stiff wire handle which passed through *K* was covered with a short piece of clay pipe stem within the finger stall. The spoon was freely movable from outside, over, and in the molten metal. It was used much like the old fashioned cream skimmer; when it was dipped into the metal and raised, the clean metal ran back through the hole, leaving the gross impurities in the spoon; these were then dumped down the exit tube *E*.

VI. Preliminary Adjustment of the Optical System.

The ordinary adjustments of the spectroscope need no description. It should be stated, however, that as the horizontal plane of the liquid reflecting surface was fixed, the optic axis of the spectroscope circle had to be brought into coincidence with it, by means of the leveling screws on the base of the instrument.

Orientation of the Plane of the Polarizer.—The ordinary means of finding this by means of reflection from a plane glass surface was found to be too inaccurate. A method was therefore devised which was quite satisfactory and is believed to be new. The compensator was removed from the instrument and the light from the polarizer, reflected from a clean pool of mercury (or other good metal surface), was examined by the analyzer. Both nicols were rotated alternately until total extinction was secured. In this case, the light coming from the metal mirror to the analyzer must have been plane polarized. Metal surfaces, however, always produce a difference of phase between the components of the reflected light in, and perpendicular to, the plane of polarization. It is evident therefore that one of these must have been absent in the reflected

and also the incident light. It is therefore also evident that the planes of polarization of the two nicols must likewise be in and perpendicular respectively to the plane of incidence of the light. In which of the two planes the light from the polarizer is polarized, can be easily determined if necessary by various simple methods which need not be discussed here.

Successive positions of the polarizer as obtained by this method always agreed within the least count of the polarizer circle, *i.e.*, 6′.

The zero phase difference readings of the compensator were found to be very slightly different for the three colors used. This instrument was also found to possess a small temperature coefficient which was negligible, however, in comparison with other sources of error.

Adjustment of Bi-Field.—This is entirely optical. With the compensator removed, the analyzer and polarizer were "crossed." The common edge of the bi-plate was set approximately perpendicular to the axis of the beam of light and the bi-plane turned until its faces made *unequal* angles with the beam. The whole bi-plate mounting was then rotated on the cap of the nicol holder until total extinction of the beam was produced *through* the plates. It should be stated, that for this work the short focus telescope previously mentioned, was used as eyepiece. The common edge now lay in the plane of polarization of the analyzer, and the mounting was therefore clamped to the cap. Next, the common edge being inclined 10° to 20° to the direction of the beam of light, the bi-plate was rotated around this edge until the intensity of the halves of the field was the same. The plates now made equal angles with the beam of light, and the bi-plate bolt was clamped to the arm on which it was supported.

VII. Experimental Procedure and Calculations.

Preparing the Surface.—After the metal had been placed in the heater and the container sealed over it, hydrogen was run through the apparatus gently for about half an hour to wash out the air. The circulation of the cooling water was then started and the furnace current turned on. As no change of the optical constants of the metals with temperature could be detected, the temperature was merely kept well above the melting point, at about 400° C.

After the first fusion, the metal was cleared of dross with the skimming spoon. Traces of oxygen from the surrounding atmosphere or surfaces in the container, seemed to act on the newly skimmed surfaces until completely exhausted. When the surface was quite clean it became extremely mobile with minute pieces of oxide floating here and there. At this stage the container was swung into place beside the spectroscope

circle and raised to the proper height, and the compensator and analyzer were roughly adjusted for minimum intensity of illumination. The surface was then examined without the telescope eyepiece. Surface impurities showed prominently as bright flecks or long streaks and were removed as far as possible by skimming. Those ultimately remaining usually travelled to the convex edge of the liquid as previously noted by Bidwell with similar surfaces. At the higher temperatures, actual reduction of the final oxide particles seemed to take place.

Readings.—These usually agreed within the limits of experimental error when made upon freshly cleaned surfaces. The surfaces of many alloys showed a progressive decrease of phase when left standing, although the analyzer readings were practically unaffected. It was therefore desirable to take the readings as rapidly as possible and the following order of taking observations was accordingly adopted:

1. The polarizer was set with its plane of polarization at 45° with the plane of incidence. Using the short focus telescope and bi-field, the analyzer was then accurately "crossed" with the reflected beam of light. Using the regular telescope eyepiece, the compensator was adjusted until the dark band in the slit image lay under the telescope cross hairs. Ten such compensator readings were taken for each of the three colors used.

2. The polarizer was turned through 90° and the compensator readings of 1 above repeated.

3. Using the bi-field and the short focus telescope as an eyepiece, five readings of the analyzer for a photometrically balanced field, for each of the three colors used, were taken.

4. The polarizer was turned back through 90° and 3 repeated.

5. 1 was repeated with yellow light to test possible change in the surface.

Owing to the perfection of the nicols and their divided circles it was found to be unnecessary to turn either polarizer or analyzer to positions approximately 180° from those already used. For positions of the polarizer 90° apart, the mean compensator readings were found to be consistently different by an amount varying from a few tenths to about 1° of phase difference. The cause of this consistent difference is unknown, but undoubtedly lies in the imperfections of the compensator.

Calculation of Optical Constants.—Mean values were taken in all calculations. The determinations of the angle of incidence and the phase difference indicated by the compensator are sufficiently obvious. The difference between the two mean analyzer positions for the two polarizer azimuths of 3 and 4 above was taken as 2ψ. It may have been the complement of 2ψ but as the substitution of this complement in the

reduction formula used to calculate the optical constants merely produced a change of sign in the values of these constants, this uncertainty gave no trouble. For metals, $\nu^2 - \kappa^2$ is always negative, and $2\nu\kappa$ positive. The reflecting power at perpendicular incidence was calculated from the formula[1]

$$R = \frac{\nu^2 + \kappa^2 + 1 - 2\nu}{\nu^2 + \kappa^2 + 1 + 2\nu}.$$

VIII. RESULTS.

The alloys used were bismuth-lead, cadmium-lead, tin-lead and bismuth-mercury. The metals were obtained from the chemical laboratory and were pure.

1. *Bismuth-lead.*—These were very satisfactory from an optical standpoint. After skimming well, the alloy surfaces remained clean indefinitely, as the hydrogen atmosphere seemed to reduce any oxide flecks remaining.

2. *Cadmium-lead.*—The cadmium evaporated quite freely from these alloys and formed bright mirrors on the observation windows in the course of an hour or so. The depolarizing effect of the films made the compensator band faint and hazy and gave the field of the bi-plate a mottled appearance, but very good readings could be taken when the mirrors were so opaque that no details of the alloy surfaces could be seen.

3. *Tin-lead.*—Work with these alloys was extremely difficult. A few seconds after the formation of a new and clean surface, fine lines of what seemed to be minute bubbles appeared and spread until the whole surface was covered with a viscous scum, probably oxide. This formation was never entirely prevented, even when the atmosphere was practically pure hydrogen. It is probably due to oxygen originally in solution in the tin. On this account, the surface of the pure tin (or of alloys rich in this component) was renewed after every two or three readings. These readings were reproducible and the phase difference indicated by the compensator was a maximum.

4. *Bismuth-mercury.*—The accuracy of results with these alloys is relatively poor. This is due to the condensation of mercury on the observation windows, and the low temperatures necessarily employed, which prevented any reducing effect by the hydrogen.

Composition of the Alloys.—As an accuracy of more than one per cent. was not necessary, simple gravimetric determinations of the composition were made. The samples obtained from each alloy were dissolved in nitric acid. Lead was estimated as sulphate in each case, bismuth as tetroxide, cadmium as nitrate, and tin as oxide. No chemical analysis

[1] Houston, A Treatise on Light, p. 417.

of the bismuth-mercury alloy was made. In the following tables the alloys will be defined by the atomic per cent. of one component.

Tables.—The experimentally determined values of the angle of incidence ϕ, twice the azimuth angle of the reflected light (2ψ), and the phase difference, Δ, are given for each of the alloys examined. From these are calculated the optical constants $\nu^2 - \kappa^2$, $2\nu\kappa$, ν, κ and R the reflecting power at perpendicular incidence.

TABLE I.

Bismuth-lead Alloys.

Alloy.		ϕ.	2ψ.	Δ.	$\nu^2-\kappa^2$.	$2\nu\kappa$.	ν.	κ.	Refl. Power, Per Cent.
				579$\mu\mu$.					
Bismuth (pure)	A	70° 32′	63° 47′	125° 48′	9.64	20.27			
	A	70° 36′	63° 37′	124° 41′	9.36	19.47			
	B	70° 35′	64° 3′	124° 41′	9.77	19.72			
Mean values.....	....	.....	.....	.....	9.62	19.72	2.48	3.97	64.4
6 per cent. lead.....	C	70° 35′	64° 36′	126° 9′	10.52	20.53			
	D	70° 35′	63° 58′	124° 38′	9.69	19.40			
Mean values.....	....	.....	.....	.....	10.10	19.97	2.52	4.00	64.5
19 per cent. lead....	E	70° 33′	64° 6′	124° 44′	9.79	19.37			
	F	70° 33′	64° 30′	126° 50′	10.51	21.06			
Mean values.....	....	.....	.....	.....	10.15	20.21	2.48	4.03	65.0
26.4 per cent. lead..	I	70° 33′	65° 11′	126° 23′	11.08	20.52	2.57	4.09	65.2
48 per cent. lead....	J	70° 33′	65° 16′	126° 14′	11.64	21.24			
	K	70° 33′	65° 36′	127° 17′	11.67	21.22			
Mean values.....	....	.....	.....	.....	11.66	21.23	2.56	4.20	65.7
60 per cent. lead....	M	70° 33′	66° 14′	127° 1′	12.24	20.85			
	N	70° 33′	65° 54′	128° 17′	12.19	22.10			
Mean values.....	....	.....	.....	.....	12.22	21.47	2.55	4.26	66.8
75 per cent. lead....	O	70° 33′	67° 28′	128° 48′	14.03	22.17			
	P	70° 33′	67° 12′	128° 14′	13.58	21.70			
Mean values.....	....	.....	.....	.....	13.81	21.94	2.62	4.37	67.4
86 per cent. lead....	Q	70° 33′	67° 23′	128° 53′	13.96	22.26	2.50	4.48	69.0
Lead (pure).....	R	70° 37′	67° 46′	129° 17′	14.63	22.74			
	S	70° 36′	68° 10′	129° 17′	15.04	22.48			
Mean values.....	....	.....	.....	.....	14.84	22.61	2.56	4.53	69.2

Alloy.		ϕ.	2ψ.	Δ.	$\left(\frac{2\pi}{\lambda}\right)^2$	$2\nu\kappa$.	ν.	κ.	Refl. Power Per Cent.
				546$\mu\mu$.					
Bismuth (pure)	A	70° 32′	63° 53′	123° 36′	9.39	18.51			
	A	70° 36′	63° 47′	123° 14′	9.25	18.26			
	B	70° 35′	63° 43′	121° 56′	8.96	17.34			
Mean values.....	....	.....	.....	.....	9.20	18.04	2.35	3.84	63.8
6 per cent. lead.....	C	70° 35′	64° 3′	123° 41′	9.54	18.53			
	D	70° 35′	64° 4′	122° 46′	9.39	17.87			
Mean values.....	....	.....	.....	.....	9.47	18.20	2.35	3.87	64.1
19 per cent. lead....	E	70° 33′	64° 2′	123° 8′	9.42	18.13	2.35	3.86	64.1
26.4 per cent. lead..	H	70° 33′	65°	125° 58′	10.80	20.17			
	I	70° 33′	65° 9′	125° 46′	10.90	19.97			
Mean values.....	....	.....	.....	.....	10.85	20.07	2.44	4.10	65.9
48 per cent. lead....	J	70° 33′	65° 26′	124° 4′	10.78	18.54			
	K	70° 33′	65° 42′	122° 2′	10.51	16.94			
Mean values.....	....	.....	.....	.....	10.65	17.74	2.24	3.96	65.2
61 per cent. lead....	L	70° 33′	66° 28′	124° 14′	11.69	18.36			
	M	70° 33′	66° 25′	126° 6′	12.13	19.97			
	N	70° 33′	66° 12′	126° 24′	12.05	20.25			
Mean values.....	....	.....	.....	.....	11.96	19.53	2.34	4.18	67.3
75 per cent. lead ...	O	70° 33′	67° 29′	126° 24′	13.30	19.94			
	P	70° 33′	66° 56′	126° 0′	12.66	19.73			
Mean values.....	....	.....	.....	.....	12.98	19.84	2.32	4.28	68.4
86 per cent. lead....	Q	70° 33′	67° 40′	127° 0′	13.65	20.33	2.33	4.37	66.1
Lead (pure).....	R	70° 36′	68° 18′	127° 29′	14.57	20.71			
	S	70° 37′	68° 18′	127° 23′	14.56	20.65			
Mean values.....	....	.....	.....	.....	14.56	20.68	2.42	4.46	69.5
				404$\mu\mu$.					
Bi.....	A	70° 32′	63° 43′	111° 48′	7.06	11.80	1.80	3.23	60.6
Pure.....									
6 per cent. lead.....	D	70° 35′	62° 50′	110° 54′	6.62	11.66	1.85	3.15	69.0
18 per cent. lead....	E	70° 33′	63° 49′	111° 54′	7.09	11.77	1.82	3.23	60.3
26.4 per cent. lead..	I	70° 33′	65° 24′	111° 54′	7.85	11.40	1.73	3.29	62.1
48 per cent. lead....	J	70° 33′	66° 1′	112° 24′	8.31	11.52			
	K	70° 33′	66° 13′	113° 54′	8.79	12.15			
Mean values.....	....	.....	.....	.....	8.55	11.84	1.74	3.40	63.5

Alloy.		ϕ.	2ψ.	Δ.	$-(n^2-k^2)$.	$2nk$.	n.	k.	Refl. Power, Per Cent.
				579$\mu\mu$.					
61 per cent. lead....	L	70° 33′	66° 56′	112° 36′	8.83	11.33			
	M	70° 33′	66° 41′	114° 6′	9.08	12.11			
	N	70° 33′	65° 32′	114° 30′	8.57	12.61			
Mean values.....	....	.....	.....	.....	8.83	12.02	1.74	3.45	64.0
75 per cent. lead....	O	70° 33′	65° 32′	115° 18′	8.77	13.02			
	P	70° 33′	67° 40′	114° 48′	9.78	12.15			
Mean values.....	....	.....	.....	.....	9.28	12.59	1.78	3.53	64.6
86 per cent. lead....	Q	70° 33′	68° 30′	115° 6′	10.30	12.01	1.66	3.61	67.0
Pure lead.....	R	70° 36′	69° 6′	116° 18′	11.11	12.48			
	S	70° 37′	69° 54′	115° 34′	11.23	11.74			
Mean values.....	....	.....	.....	.....	11.17	12.11	1.63	3.72	68.5

Table II.

Cadmium-lead Alloys.

Alloy.		ϕ.	2ψ.	Δ.	$-(n^2-k^2)$.	$2nk$.	n.	k.	Refl. Power, Per Cent.
				579 $\mu\mu$.					
Cadmium (pure) ...	A	70° 37′	82° 38′	126° 36′	25.66	8.78			
	B	70° 37′	83° 56′	127° 0′	26.74	7.49			
	C	70° 37′	82° 8′	127° 6′	25.99	9.61			
	D	70° 37′	82° 56′	126° 0′	25.19	8.19			
Mean values.....	....	.....	.....	.....	25.72	8.15	0.82	5.14	89.8
12 per cent. lead....	F	70° 37′	81° 22′	126° 42′	25.14	10.23	1.00	5.11	86.7
22 per cent. lead....	H	70° 36′	77° 23′	127° 42′	23.30	14.88			
	I	70° 36′	77° 54′	127° 0′	23.06	13.86			
Mean values.....	....	.....	.....	.....	23.19	14.37	1.43	5.02	81.6
33.5 per cent. lead..	J	70° 36′	75° 43′	128° 24′	22.46	16.96	1.69	5.03	79.3
52.5 per cent. lead..	K	70° 36′	73° 53′	128° 12′	20.56	18.24	1.86	4.90	76.2
64 per cent. lead....	L	70° 36′	72° 26′	127° 30′	18.98	19.03	1.99	4.79	75.0
76 per cent. lead....	M	70° 36′	69° 58′	127° 54′	16.44	20.36	2.21	4.62	72.0
79.5 per cent. lead..	N	70° 36′	69° 40′	128° 15′	15.59	20.00	2.21	4.53	71.2
Pure lead.....	....	.....	.....	.....	14.84	22.61	2.56	4.53	69.2

Alloy.		ϕ.	2ψ.	Δ.	$-(n^2-\kappa^2)$.	$2n\kappa$.	n.	κ.	Refl. Power, Per Cent.
				546 $\mu\mu$.					
Cadmium (pure)....	A	70° 37′	82° 18′	123° 24′	22.32	7.32			
	B	70° 37′	83° 56′	124° 0′	23.52	6.33			
	C	70° 37′	82° 11′	124° 10′	22.81	8.07			
	D	70° 37′	82° 40′	124° 12′	23.20	7.68			
Mean values.....	....	.....	.....	... ...	23.16	7.45	0.76	4.87	88.6
12 per cent. lead....	F	70° 37′	81° 24′	123° 36′	22.09	8.64	0.90	4.79	86.4
22 per cent. lead....	H	70° 36′	77° 18′	125° 16′	21.14	13.16			
	I	70° 36′	77° 54′	125° 40′	21.02	12.57			
Mean values.....	....	.....	.....	.....	21.08	12.87	1.14	4.78	83.4
33.5 per cent. lead..	J	70° 36′	75° 48′	126° 36′	21.07	15.38	1.58	4.86	79.0
52.5 per cent. lead..	K	70° 36′	74° 4′	126° 4′	19.25	16.25	1.72	4.71	76.7
64 per cent. lead....	L	70° 36′	72° 13′	125° 17′	16.17	17.11	1.92	4.46	72.2
76 per cent. lead....	M	70° 36′	70° 10′	125° 37′	15.52	18.14	2.04	4.44	71.1
79.5 per cent.....	N	70° 36′	70° 4′	125° 29′	15.38	18.10	2.05	4.42	71.6
Pure lead.....	....	.....	.....	.....	14.56	20.68	2.42	4.46	69.5
				404 $\mu\mu$.					
Cadmium (pure)....	A	70° 37′	82° 12′	109° 6′	12.62	4.00			
	B	70° 37′	85° 23′	107° 18′	12.16	2.22			
	C	70° 37′	82° 11′	109° 27′	12.79	4.07			
Mean values.....	....	.....	.....	.....	12.52	2.76	0.39	3.56	93.1
12 per cent. lead....	E	70° 37′	82° 25′	108° 6′	12.17	3.73			
	F	70° 37′	82° 12′	109° 54′	13.01	4.14			
Mean values.....	....	.....	.....	.....	12.59	3.94	0.55	3.59	85.6
22 per cent. lead....	H	70° 36′	78° 28′	111° 48′	13.11	6.37			
	I	70° 36′	80° 0′	112° 6′	13.63	5.73			
Mean values.....	....	.....	.....	.....	13.37	6.05	0.81	3.74	81.4
33.5 per cent. lead..	J	70° 36′	77° 0′	113° 42′	13.57	7.78	1.02	3.82	78.2
52.5 per cent. lead..	K	70° 36′	75° 5′	113° 15′	12.65	8.58	1.15	3.74	75.2
64 per cent. lead....	L	70° 36′	73° 10′	112° 54′	11.74	9.30	1.27	3.66	72.4
76 per cent. lead....	M	70° 36′	70° 40′	113° 6′	9.56	10.47	1.52	3.45	66.6
79.5 per cent. lead..	N	70° 36′	70° 2′	113° 48′	10.71	10.94	1.52	3.61	68.6
Pure lead.....	....	.....	.....	.....	11.17	12.11	1.63	3.72	68.5

Table III.

Tin-lead Alloys.

Alloy.		ϕ.	2ψ.	Δ.	$\frac{(n^2-k^2)}{(n^2+k^2)^2}$	$2nk$.	n.	k.	Refl. Power, Per Cent.
				579$\mu\mu$.					
Tin, pure	A	70° 33′	76° 38′	131° 40′	26.24	19.21			
	H	70° 41′	76° 55′	131° 54′	27.24	19.47			
Mean values					26.74	19.34	1.77	5.47	81.1
7 per cent. lead	I	70° 41′	77° 22′	131° 3′	24.68	20.12	1.89	5.32	79.3
21 per cent. lead	B	70° 33′	75° 40′	131° 48′	25.28	20.32	1.89	5.37	79.7
37.5 per cent. lead	C	70° 33′	73° 38′	130° 30′	21.83	20.62	2.03	5.09	76.9
62 per cent. lead	D	70° 33′	71° 18′	129° 37′	18.67	21.28	2.20	4.85	73.9
85.5 per cent. lead	E	70° 36′	70° 47′	129° 30′	17.86	21.46	2.24	4.78	73.1
Pure lead		—			14.84	22.61	2.56	4.53	69.2
				546$\mu\mu$.					
Tin, pure	A	70° 33′	76° 4′	131° 27′	25.38	19.53			
	H	70° 41′	77° 13′	130° 7′	25.68	17.32			
Mean values					25.53	18.43	1.73	5.34	80.8
7 per cent. lead	I	70° 41′	76° 22′	129° 11′	24.00	17.27	1.70	5.18	80.3
21 per cent. lead	B	70° 33′	74° 47′	127° 53′	22.15	18.48	1.83	5.05	78.1
37.5 per cent. lead	C	70° 33′	73° 10′	127° 53′	19.54	18.36	1.91	4.81	75.9
62 per cent. lead	D	70° 33′	71° 18′	127° 20′	17.17	18.78	2.04	4.62	73.3
85.5 per cent. lead	E	70° 36′	71° 24′	126° 36′	17.20	18.40	2.00	4.60	73.5
Pure lead					14.56	20.68	2.42	4.46	69.5
				404$\mu\mu$.					
Tin, pure	A	70° 33′	76° 41′	116° 53′	15.03	9.09			
	H	70° 41′	75° 29′	117° 13′	14.93	10.07			
Mean values					10.98	9.58	1.18	4.05	77.6
7 per cent. lead	I	70° 41′	74° 58′	116° 13′	14.19	9.92	1.25	3.97	76.0
21 per cent. lead	B	70° 33′	76° 32′	117° 42′	15.42	9.52	1.16	4.10	78.3
37.5 per cent. lead	C	70° 33′	74° 44′	116° 42′	13.22	9.51	1.24	3.84	74.9
85.5 per cent. lead	F	70° 36′	72° 7′	115° 30′	12.37	11.11	1.46	3.81	71.5
Pure lead					11.17	12.11	1.63	3.73	68.5

TABLE IV.

Bismuth-mercury Alloys.

Alloy.	ϕ.	2ψ.	Δ.	$-(\mu^2-\kappa^2)$.	$2\mu\kappa$.	ν.	κ.	Refl. Power, Per Cent.
579$\mu\mu$.								
Bismuth, pure				9.70	19.72	2.48	3.97	64.4
Bi-Hg (40 per cent. Hg?)	70° 36′	64° 36′	124° 6′	10.32	19.11	2.38	4.00	67.7
Mercury, pure (Meier's results)				16.7	14.2	1.62	4.39	75.3
546$\mu\mu$.								
Bismuth, pure				9.20	18.04	2.35	3.84	63.8
Bi-Hg (40 per cent. Hg?)	70° 36′	65° 40′	122° 54′	10.92	17.91	2.24	3.99	66.1
Mercury, pure (Meier's results)				15.2	12.3	1.48	4.17	74.9
404$\mu\mu$.								
Bismuth, pure				7.06	11.80	1.80	3.23	60.6
Bi-Hg (40 per cent. Hg?)	70° 36′	67° 2′	110° 3′	8.33	10.44	1.59	3.29	63.6
Mercury, pure (Meier's results)				9.2	5.8	0.92	3.17	73.3

TABLE V.

Ne^2/m and *W*.

Bismuth-lead Alloys.

	Per Cent. Lead.	0.	6.	19.	26.4.	48.	61.	75.	86.	100.
$Ne^2/m \times$ const.	579 $\mu\mu$	131	132	134	133	137	137	137	139	143
	546 $\mu\mu$	133	133	132	146?	124?	137	137	137	142
	404 $\mu\mu$	148	147	149	141	146	144	152	147	149
$W \times$ const.	579 $\mu\mu$	281	274	276	261	259	252	232	234	223
	546 $\mu\mu$	283	278	279	276	247	247	235	233	223
	404 $\mu\mu$	308	323	309	275	268	265	267	234	220

Cadmium-lead Alloys.

	Per Cent. Lead.	0.	12.	22.	33.5.	52.5.	64.	76.	79.5.	100.
$Ne^2/m \times$ const.	579 $\mu\mu$	92	94	101	105	112	114	122	120	140
	546 $\mu\mu$	93	93	103	115	114	113	122	121	141
	404 $\mu\mu$	95	99	115	122	123	125	129	137	149
$W \times$ const.	579 $\mu\mu$	52	64	97	105	123	152	186	121	223
	546 $\mu\mu$	51	65	100	123	137	156	184	186	223
	404 $\mu\mu$	46	64	92	110	140	162	214	204	229

Tin-lead Alloys.

	Per Cent. Lead.	0.	7.	21.	37.5.	62.	85.5.	100.		
$Ne^2/m \times$ const.	579 $\mu\mu$	125	123	127	125	127	125	143		
	546 $\mu\mu$	134	127	129	125	126	124	141		
	404 $\mu\mu$	136	150	132	150	——	140	150		
$W \times$ const.	579 $\mu\mu$	114	123	126	147	173	179	223		
	546 $\mu\mu$	120	119	138	151	175	171	223		
	404 $\mu\mu$	136	150	132	150	——	182	220		

Bismuth-mercury Alloys.

	$\frac{Ne^2}{m} \times$ const.			$W \times$ const.		
	Bismuth.	*Bi — Hg.*	Mercury.	Bismuth.	*Bi — Hg.*	Mercury.
579 $\mu\mu$	131	127	92	281	257	135
546 $\mu\mu$	133	125	88	283	245	127
404 $\mu\mu$	148	129	89	308	242	123

The observational errors inherent in katoptric methods give possible variations of 3 or 4 per cent. in the values of ν and κ, and correspondingly larger variations in $\nu^2 - \kappa^2$ and $2\nu\kappa$. These limits suppose perfect metal surfaces. From an inspection of the values obtained by the writer for different surfaces of the same liquid surface it will be seen that this limit is practically attained. As observations upon different surfaces of solid metals of the same composition differ among themselves by 10 to 30 per cent. in some cases, even for the same observer, the experimental advantages of using liquid surfaces are manifest. There are no scratches or surface impurities due to abrasive material, no differential polishing of the components of an alloy, and no question of variation of surface due to heat treatment. The surfaces are always plane and can be renewed at pleasure.

The determinations made with violet light (404 $\mu\mu$) were exceedingly difficult to make on account of eye-fatigue. The values of the optical constants for this light are therefore regarded merely as a check on the other determinations.

IX. Discussion of Results.

Interpretation of the above results is most conveniently made on the simple free electron theory of Drude. His conception of the free electrons ricochetting from molecule to molecule under the influence of an outside applied electromagnetic field is inadequate, especially in the light of the more recent developments of the electron theory. Nevertheless, his

classic treatment is capable of explaining and coordinating metallic phenomena qualitatively with surprising success.

Drude's[1] equations for the optical constants of the metals are as follows:

$$n^2(1 - k^2) = 1 + \Sigma \frac{\theta_h'}{1 - \left(\frac{\tau_h}{\tau}\right)^2} - 4\pi \Sigma \frac{m'N}{r^2 + \left(\frac{m'}{\tau}\right)^2}, \qquad (1)$$

$$2n^2k = 4\pi\tau\Sigma \frac{Nr}{r^2 + \left(\frac{m'}{\tau}\right)^2}. \qquad (2)$$

The first summation in equation (1) is a term wholly due to the molecules of the metal. The other two summations in both equations express the optical effect of the free electrons, being stated wholly in terms of their parameters and τ, the period of vibration of the light used, divided by 2π. In these latter summations, m' is equal to m/e^2, m and e being the mass and charge of each of any group of free electrons. N is the number per cm^3. of each group. r is equal to N/σ, σ being that portion of the total "conductivity" of the metal due to these particular electrons. σ should be equal to the electrical conductivity as usually defined for steady currents.

In this work, the following simplifying assumptions are usually made:

(*a*) In equation (1), the terms $1 + \Sigma \cdot \theta_h'/[1 - (\tau_h/\tau)^2]$ have been assumed to possess a value independent of the free electrons present, *i.e.*, these terms have a numerical value equal approximately to that of $n^2(1 - k^2)$ for a heavy insulator. As selective reflection is very small for the metals used in this investigation, n may be taken as about 1.6 and k as 0. Thus $n^2(1 - k^2)$ for the molecules of the metal, or its equivalent $1 + \Sigma \cdot \theta_h'/[1 - (\tau_n/\tau)^2]$ will be numerically equal to about 2.5. Since $n^2(1 - k^2)$ is a much larger negative quantity for metals, rather wide variations for this assumed value of 2.5 will not seriously affect the final results.

(*b*) The free electrons are assumed to be of one group, *i.e.*, they are negative electrons possessing identical mass, charge, average velocity, etc. The positive electrons are essentially molecules possessing positive charges. Being relatively large and heavy, they contribute practically nothing to the electrical and optical characteristics peculiar to metals. Their optical effect is thus entirely contained in the first summation of equation (1) given in (*a*).

If we divide numerators and denominators of the fractions in (1) and

[1] P. Drude, Theory of Optics (Mann's translation), p. 398

(2) by m'^2 and substitute m/e^2 for m', the above equations become

$$\nu^2 - \kappa^2 = 2.5 - \frac{4\pi \dfrac{Ne^2}{m}}{\dfrac{r^2e^4}{m^2} + \left(\dfrac{1}{\tau}\right)^2}, \tag{3}$$

$$2\nu\kappa = 4\pi\tau \frac{\dfrac{Ne^2}{m}}{\dfrac{r^2e^4}{m^2} + \left(\dfrac{1}{\tau}\right)^2}, \tag{4}$$

where ν and κ have been used for Drude's n and $n\kappa$ respectively. r has been given as equal to N/σ. σ, the electrical conductivity, is expressible in the free electron theory of Drude by $Ne^2/m \cdot T/2$, in which T is the average value of the time between successive impacts of a free electron with successive molecules. This period is equal to the reciprocal of the number of impacts of the electron with molecules per second, and will be the same for all electrons. If we substitute $1/W$ for $T/2$, σ is equal to Ne^2/mW. W is thus a sort of quasi-frequency of impact of the free electrons. r is now given by mW/e^2. If this is substituted for r in equations (3) and (4), and ω for $1/\tau$ (*i.e.*, ω for 2π times the frequency of the light used), these equations become

$$\nu^2 - \kappa^2 = 2.5 - \frac{4\pi \dfrac{Ne^2}{m}}{\omega^2 + W^2}, \tag{5}$$

$$2\nu\kappa = 4\pi \frac{W}{\omega} \frac{\dfrac{Ne^2}{m}}{\omega^2 + W^2}. \tag{6}$$

W and Ne^2/m may be computed from these equations and the values of the optical constants. The reasons for the choice of W rather than T as a free electron parameter will perhaps be evident in the discussion following.

Optical Constants in the Solid and Liquid States.—Figures 3, 4, 5 and 6 show the relations between the optical constants for the solid and liquid states of the pure metals. The data for the solid metals has been obtained from various papers. It is seen that with the exception of cadmium, these constants are roughly doubled by fusion. Figs. 7, 8 and 9 show the variation of the constants $\nu\kappa$ and $\nu^2 - \kappa^2$ for liquids, with the atomic concentration of one component. In general, these are smooth, nonlinear curves. Littleton has assumed that simple mixture in solid alloys will be shown by the straight line variation of these con-

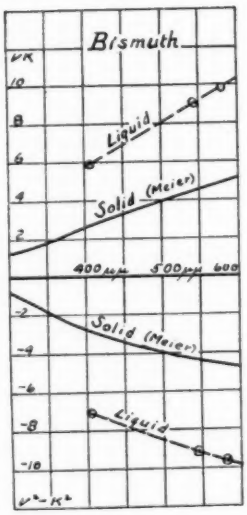


Fig. 3.

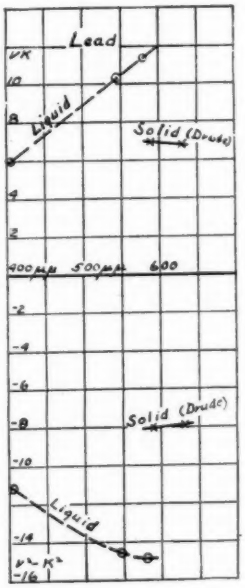


Fig. 4.

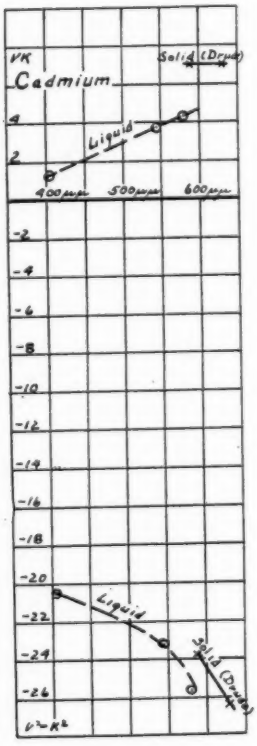


Fig. 5.

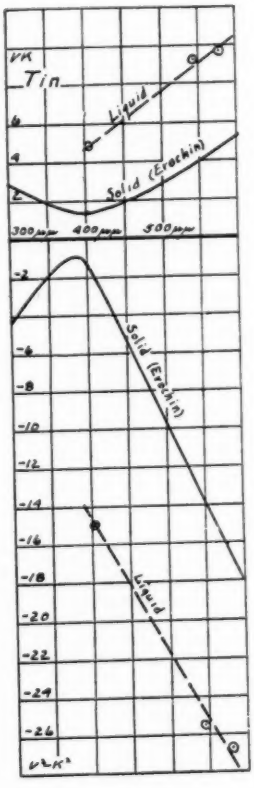


Fig. 6.

stants with the composition. This, of course, will be so where the alloy consists of two types of crystals mechanically mixed.

Theoretical Interpretation of Results.—In Table V. the calculated values of Ne^2/m and W are given for all the pure metals and alloys investigated. The units used are arbitrary. In Figs. 10, 11 and 12 they have been plotted against the atomic concentration of one component, for each series of alloys respectively.

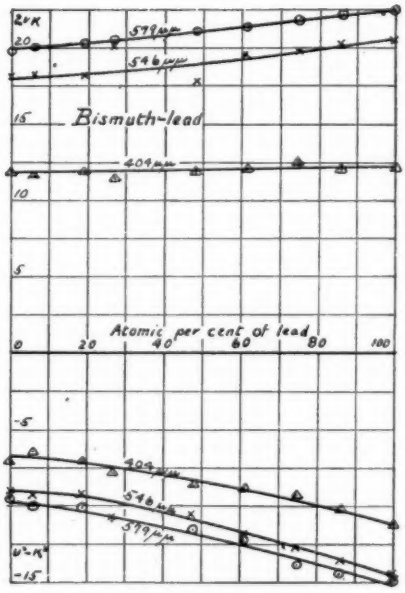


Fig. 7.

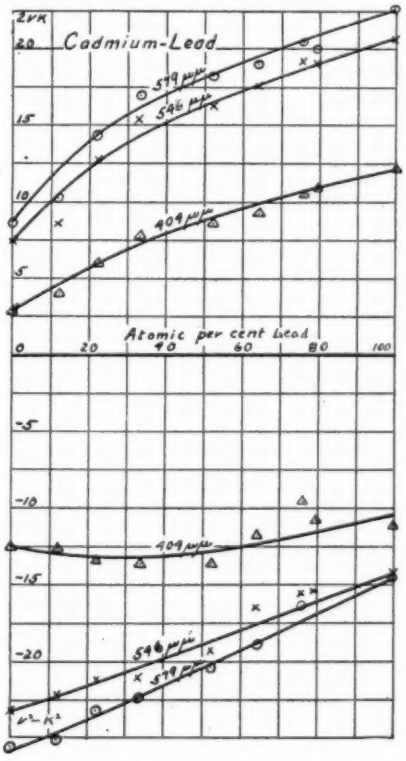


Fig. 8.

Within the limits of error, the values of Ne^2/m calculated from the optical constants for yellow and green light (579 $\mu\mu$ and 546 $\mu\mu$) seem to lie on the same straight line for each series, except for the pure lead points in Figs. 11 and 12. While the results for violet light (404 $\mu\mu$) are altogether too uncertain to be used in determining any functional relationship of Ne^2/m, there seems to be no doubt that in general they lie above the other points. Ne^2/m, however, should be the same, whatever the wave-length of the light used in determining it. The discrepancy is probably due to the following causes: (*a*) the effect of atomic resonance may not be even approximately represented by 2.5; (*b*) selective reflec-

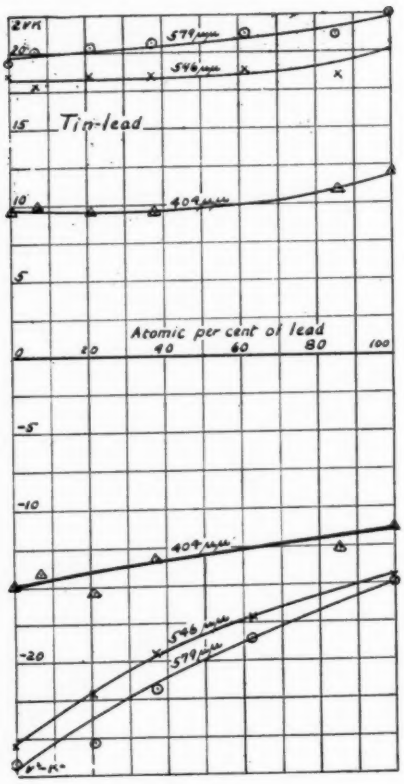


Fig. 9.

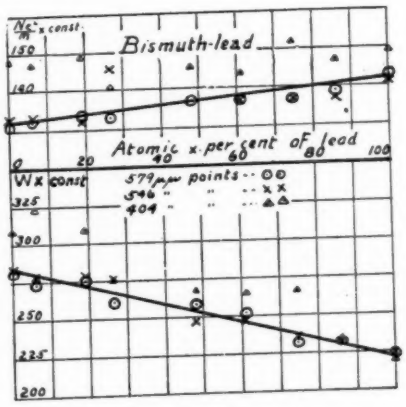


Fig. 10.

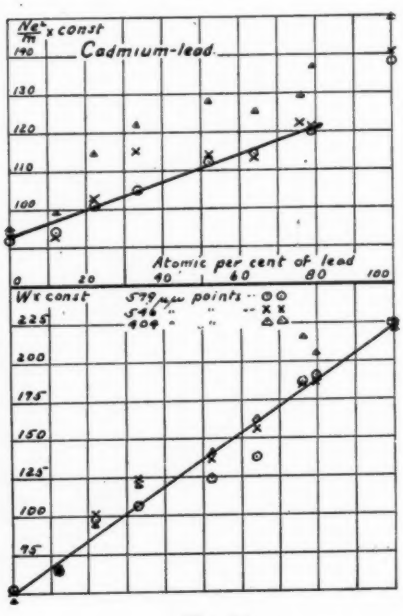


Fig. 11.

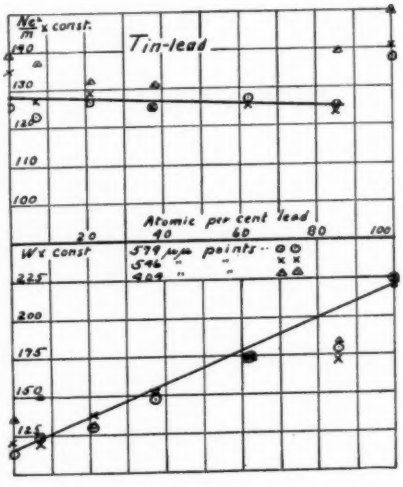


Fig. 12.

tion may be present in the ultra violet, not far beyond the visible. This is the case for most solid insulators. As selective reflection is due to the resonance of bound electrons within the atom, its presence in the atoms of most metals may be inferred. The above effect may be indicated for bismuth by the slight irregularities at the violet end of the solid curves for this metal. In the case of tin, Erochin has placed the center of an absorption band at 340 $\mu\mu$. Selective reflection in the ultra-violet usually increases the value of the term in the expression of $\nu^2 - \kappa^2$ for violet light, which represents the effect of the bound electrons of the atoms, and will introduce a somewhat similar term into the expression of $2\nu\kappa$.

The probable linearity of the variation of Ne^2/m with atomic concentration is to be expected. The molecules of a liquid metal are distinct entities, practically independent of each other. The fields of force of their component electric charges are thus almost entirely internal. The conducting electrons may therefore be regarded as truly "free," and as a definite portion of the metal. Thus each component of the alloy carries to the alloy a certain number of free electrons, identical in characteristics, and the number of these in the mixture is the sum of the numbers present in each component before the alloy is formed.

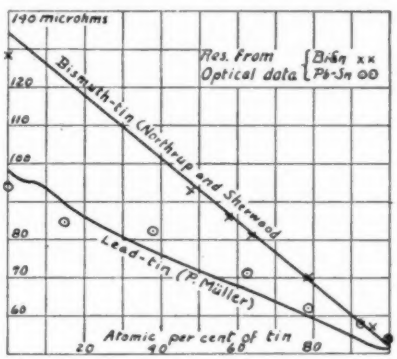


Fig. 13.

The pure lead points in the Cd-Pb and Sn-Pb figures are not collinear with the other points, and the difference can hardly be ascribed to experimental error. The cause of this variation, therefore, probably lies in some peculiarity of constitution of these alloys. Such a peculiarity is indicated by the resistances of this series of alloys, as given by P. Müller.[1] In the tin-lead series, the resistance rises uniformly with the lead content up to about 90 per cent. lead. Beyond this the resistance curve is lifted up in a small but distinct maximum (see Fig. 13). Müller gives

[1] P. Muller, Metallurgie, 7, 755, 1910.

the resistances of the cadmium-lead series to 6 per cent. cadmium only, but the same rise is evident. The fall of resistance from the maximum to pure lead is probably due to the anomalous increase in the number of free electrons indicated by the optical results.

In the solid state, these alloys are solid solutions at the lead side of the phase diagrams. The temperature of 400° at which the observations were made, is not far above the temperature of complete fusion, and it is quite possible that the tendency to atomic grouping shown by the solutions is carried over into the molten state. This tendency is analogous to the formation of an easily dissociated compound, and the process of grouping will result in a similar manner in the absorption of free electrons into the groups. Thus the alloy from 0 per cent. to 90 per cent. lead may be considered as a simple mixture of cadmium (or tin) and this quasi-compound, and the atomic concentration—Ne^2/m curve will be a straight line within this range. The alloy series from 90 per cent. to 100 per cent. lead should act as a mixture of the so-called compound and pure lead, in a similar manner.

Turning to the curves which show the variation of W with the atomic concentration, the agreement of the values of W given by the three lights used is very good, except for alloys rich in bismuth. The differences for this metal series may be due to either or both of the causes given for the Ne^2/m variations. It should be possible to bring the values of both Ne^2/m and W into good agreement for the three lights used, by assuming suitable values of the molecular or bound electron terms in the optical equations. However, in view of the rather large errors inherent in optical measurements on metals, it has seemed hardly worth while to do this.

The linearity of the W curves is striking, and is also a natural consequence of the liquid condition of the alloys. The frequency of impact of a free electron with the comparatively inert molecules ($2W$) is proportional to the "total surface of impact" of the molecules. If we introduce into the assemblage of like molecules of the one metal molecules of another metal which are of a different size, the "total surface of impact," and therefore the frequency of impact of a free electron, are both obviously linear functions of the atomic concentration of either component.

It should be noticed that there is no indication in these curves of any peculiarity of cadmium-lead and tin-lead alloys rich in lead, such as that shown by the variation of Ne^2/m. The tendency towards atomic grouping which would account for this variation cannot be very great. This is shown in P. Müller's work. As the temperature is increased the irregularities of the concentration-resistance curves of these alloys dis-

appear. Owing to this looseness of grouping there is no marked shielding of one atom by another. This will be especially true if the targets of impact are the atomic nuclei. Under these conditions the frequency of impact of any free electron will be unaffected by such tendencies, and the variation of W with atomic concentration will be linear. If true compounds were formed, however, the close association of atoms might be expected to influence this free electron parameter.

Thus the variation of the free electron functions, Ne^2/m and W in these binary alloys and the coherence of results, seem to confirm very strongly the admissibility of the application of the simple form of the electron theory to liquid metals. However, any satisfactory optical theory should be able to show a quantitative relation between the electrical and optical characteristics of metals. Any of these characteristics or any of the parameters of the conducting electrons, should be determinable from a sufficient number of the remainder. For instance, Drude,[1] Schuster,[2] W. Meier[3] and others have calculated the number of free electrons per atom in various solid metals from the optical constants and electrical conductivity. Rubens and Hagen[4] tested the relationship of the electrical conductivity and reflecting power of metals given in the equation $1 - R = 2/\sqrt{\sigma T}$, in which R is the reflecting power, σ the electrical conductivity and T the frequency of the light used. This equation is not valid for visible light but holds very well for the infra-red (25.5 μ) with one or two exceptions. These exceptions have been held to be due to the neglect of "resonance of the molecules" in the electromagnetic theory of Maxwell, from which the equation is derived. Now Drude's equations, forms of which are used in this paper, are recognized as a better representation of the optical behavior of metals, than Maxwell's original equations. $1 - R = 2/\sqrt{\sigma T}$ may be derived from Drude's equations if we assume W very large in comparison with ω, *i.e.*, that the time between successive impacts of an electron may be neglected in comparison with the period of the light used in the optical determinations. Thus the lack of agreement of Maxwell's equation with facts in the case of bismuth and one or two other metals is probably due to the omission of a term which has nothing to do with "resonance of the molecules."

In general it may be said that when solid metals are used, the data obtained from electrical measurements agree rather poorly with those from optical measurements in the visible spectrum. This may be illus-

[1] P. Drude, Ann. d. Phys., 14, 936, 1904.

[2] A. Schuster, Phil. Mag., 7, 151, 1904.

[3] W. Meier, Ann. d. Phys., 31, 1016, 1910.

[4] Rubens and Hagen, Ann. d. Phys., 11, 873, 1903.

trated very well for the solid state of the metals which were investigated in this work. From equations (5) and (6) we may obtain

$$\frac{Ne^2}{m} = -\frac{\omega^2}{4\pi}\frac{(\nu^2 - \kappa^2 - 2.5)^2 - (2\nu\kappa)^2}{(\nu^2 - \kappa^2 - 2.5)}$$

and

$$W = -\frac{\omega 2\nu\kappa}{(\nu^2 - \kappa^2 - 2.5)}.$$

The electrical restivity (in electrostatic units) is Wm/Ne^2 and substituting from the equations above,

$$\text{Res. (e.s.u.)} = \frac{4\pi}{\omega}\frac{2\nu\kappa}{(\nu^2 - \kappa^2 - 2.5)^2 - (2\nu\kappa)^2}.$$

I have used the data of the various writers from which the solid metal curves of Figs. 1, 2, 3 and 4 were drawn to calculate the restivities. These are expressed in microhms and compared with the electrical resistivities of the same solid metals as given by the Smithsonian Tables, as follows:

	Bismuth.	Lead.	Cadmium.	Tin.
Light used	600 $\mu\mu$	600 $\mu\mu$	560 $\mu\mu$	550 $\mu\mu$
Res. (microhms) by opt. methods	238	159	58	76
Res. (microhms) by Smiths. Tables	119	22	7.5	14

The lack of agreement indicated above has been explained on the supposition that the resistivity at frequencies approaching that of light is markedly different from that at ordinary frequencies. Livens[1] and others have developed this functional relation. Owing to the simple conditions existing in molten metals, it should be possible to use their optical constants to gain considerable information as to this possible variation of resistivity, and as to the identity of the electrical and optical parameters given in the elementary equations. On this account I have calculated the resistivities of the pure molten metals from the optical data I have obtained. These and various electron parameters are given in the following table. Only the optical results for yellow and green light were used, those for violet light having altogether too large experimental errors. Meier's[2] results were used for mercury. The resistivities so obtained, are compared with those found by Northrup and Suydam[3] for bismuth, lead, cadmium and tin at 400°. While no particular effort

[1] G. H. Livens, Phil. Mag., 30, 112, 1915.

[2] W. Meier, *loc. cit.*

[3] E. F. Northrup and V. A. Suydam, Journ. Franklin Inst., 175, 153, 1913.

was made to keep the temperature of the liquid metals constant during the optical investigation, it was never very far from this value. The resistivity of mercury at ordinary temperatures was used.

TABLE VI.

	Bismuth.	Lead.	Cadmium.	Tin.	Mercury (Meier).
$Ne^2/m \times 10^{-28}/9$	132	142.5	92.5	129.5	90
N (No. free electrons per cm.^{3}) $\times 10^{-22}$	14.7	15.9	10.3	14.5	10.0
No. atoms per cm.$^3 \times 10^{-22}$	2.9	3.1	4.3	3.5	4.1
No. free electrons per atom	5.1	5.1	2.4	4.1	2.5
$W \times 10^{-12}/6\pi$	282	223	51.2	117	131
Impacts of free electron per sec. $(2W) \times 10^{-14}$	106	84	19.5	44	50
Impacts per period of green light (550 $\mu\mu$)	19.3	15.3	3.5	8.9	9.0
Wm/Ne^2 (resistivity, e.s.u.) $\times 6.60 \times 10^{-7}$	2.14	1.57	0.56	0.90	1.46
Resistivity in microhms from above data	128	94	33.4	54	87.3
Resistivity in microhms from electrical data	134	98	34	52	94
Ratio "optical" to electrical resistivity	.95	.96	.98	1.04	.93

An auxiliary figure (Fig. 13) is also given which compares the resistivities of tin-lead and tin-bismuth alloys, electrically and optically determined. The electrical data for the two series of alloys are taken from papers by P. Müller,[1] and by E. F. Northrup and R. G. Sherwood[2] respectively. The "optical" resistances of the tin-lead alloys were taken directly from the optical results of this series. As the optical characteristics of tin-bismuth alloys were not investigated, the "optical" resistances of this series were calculated from the electron parameters Ne^2/m and W of the pure metal. Both of these are linear functions of the atomic concentration, provided that no compounds are present in the liquid state, and may be easily determined for an alloy of the two metals of any given atomic composition. The quotient of these two will give the resistivity.

From an inspection of the tables and auxiliary figures, it may be seen that, except for mercury, the electrically and optically determined resistivities agree, in every case, within the limits set by unavoidable experimental error. The optical constants of mercury were not determined by the writer and their accuracy is not known, although it is probably good. There is no evidence of any change in the electrical resistivity, even up to the frequencies of light. Certainly any such variation is not greater than 5 per cent. The agreement of the results, both electrical and optical is a strong confirmation of the existence of essentially "free" electrons in these liquid metals. In view of the

[1] P. Müller, *loc. cit.*

[2] E. F. Northrup and R. G. Sherwood, Journ. Franklin Inst., 182, 477, 1916.

probable validity of this conception, I have calculated the values of N, the number of free electrons per cm.^{3}, the number per atom, the frequency of impact of the free electrons ($2W$) and the ratio of this frequency to that of green light (550 $\mu\mu$). The number of free electrons per atom necessitated a knowledge of the number of atoms per cm.^{3} For each metal this number is equal to Avogadro's number multiplied by the density and divided by the atomic weight. Avogadro's number was taken as 60×10^{22}. The densities of the molten metals are not accurate, as the temperatures at which they were taken were not given in the tables consulted. To calculate the number of electrons per cm.^{3}, e was taken as equal to 4.77×10^{-10} e.s.u. and m to 9×10^{-28} g. It is evident that the uncertainties of the various constants used in these calculations render the values of the number of free electrons per atom only approximations at best. In addition it is probable that the parameters Ne^2/m and W given here, in reality include constant multipliers. These would be introduced in a more exact development of the free electron theory.

From certain points of view it would seem that the number of free electrons per atom, or at least per molecule, should be an integer for each metal. The accuracy of the present work is not sufficient to settle this point. On the other hand, the uncertainties of the constants used to obtain these relations, affect all alike, and it is possible to obtain these ratios as integers, within the limits of experimental error, by making small and legitimate changes in the above doubtful constants. The results are certainly not inconsistent with the possibility of these integral relations.

From the agreement of the electrical and optical data in Table VI., it seems that changes of resistivity at light frequencies, and molecular resonance (except in infrequent cases) are negligible factors in determining the phenomena of these liquid metals. I can see no reason why they should be of any greater importance in solid metals, and the discrepancies encountered in the explanation of the optical and electrical phenomena of the solid phase must be due to some different cause. The distinguishing characteristic of the solid metal is the space lattice structure of its crystals. This structure demands that the atoms should have strong external fields of force, binding them into a rigid form. The conducting electrons can hardly wander freely in inter-atomic space, between so-called impacts. From various points of view it is hard to avoid the conclusion that conduction in crystalline solids is due to the "handing along" of these electrons from atom to atom. Thus, in the fundamental equations of motion of a conducting electron in the metallic crystal, a term must be present which is a function of the position of the electron within

the space lattice. It is probable that the neglect of such a term is the source of apparently discordant electrical and optical results for solid metals.

The small range of temperature and the limited accuracy of the experimental work made impossible the detection of measurement of any temperature effect on the optical constants. Such measurement would be of value from a theoretical standpoint.

PHYSICAL LABORATORY,
UNIVERSITY OF MICHIGAN.

ON THE MAINTENANCE OF VIBRATIONS OF WIRES BY ELECTRICAL HEATING.

BY N. C. KRISHNAIYAR.

SYNOPSIS.

The amplitude and phase of the vibrations set up in a stretched wire when a single phase alternating current of 50 cycles per second was sent through the wire have been investigated experimentally, and the results have been compared with the theory given by C. V. Raman.

Amplitude.—(*a*) Sonometer nearly vertical, length of vibrating segment constant, and tension varied by steps. When the tension appreciably exceeded the value corresponding to the free-period of the wire no maintained vibrations were established when the current was turned on. As the tension was decreased, persistent vibrations of small amplitude were set up when the resonance tension was approached. The amplitude progressively increased as the tension was decreased through, and some interval below, the resonance tension. Eventually a critical tension was reached at which there was a sudden complete cessation of maintenance. On reversing the process, by gradually increasing the tension, persistent vibrations were not set up unless the wire was strongly plucked at its middle. (*b*) Sonometer horizontal, tension constant, and length of wire decreased progressively. The phenomena observed were essentially the same as in the preceding case.

Phase.—By adapting the principle of the vibration microscope it was observed that the phase of the maintained vibration relatively to the phase of the current was not the same over the whole range of persistent vibrations. The phase changed rapidly by about 90° near the end of the range corresponding to very small amplitudes. For larger amplitudes and over the greater part of the range the phase remained practically constant.

Theory.—When the differential equation of the motion is solved to the first approximation it is found that the square of the amplitude of the vibrations is a linear func- of the difference of the squares of the forced and free frequencies. Also the phase should be constant. The experimental curves agree with the theoretical parabolas over all of the range except the portion pertaining to small amplitudes. A like statement applies to the phase. Further work is in progress to throw light on the observed points of inflection, and to account for the discrepancies between theory and experiment.

I. INTRODUCTION.

WHEN an intermittent electric current traverses a fine wire, the passage of the current may result in setting the wire in vibration. This phenomenon was described some years ago by Argyropoulos.[1] Recently, the analogous phenomenon obtained by sending an alternating electric current through a freely hanging platinum wire has been studied

[1] Comptes Rendus, 111 (1890), 525.

by Strientz,[1] who observed specially striking effects when the wire was heated to incandescence by the current. The present author has carried out a study of the vibrations produced by sending a single-phase alternating current of fifty cycles per second through fine wires of adjustable length or tension stretched on a sonometer, the special object of the investigation being to ascertain the manner in which the amplitude and phase of the maintained vibration vary with the difference between the natural frequency of vibrations of the wire and the frequency of the alternating current. This inquiry appeared to be called for in view of the peculiar character of the maintenance of vibration in this case. The passage of an alternating current through a wire under tension does not *directly* tend (in the absence of any external magnetic field[2]) to displace it from its position of equilibrium. The production of vibration is therefore an indirect effect due to the fact that the passage of the alternating current heats the wire and thus causes a periodic variation of tension of double the frequency of the current, and this in turn sets up and maintains a vibration having the *same* frequency as the current. In a paper[3] on "Some Remarkable cases of Resonance," published recently in this REVIEW, C. V. Raman has discussed the theory of the maintenance of vibrations by forces of double frequency and has shown that the variation of the amplitude and phase of the maintained vibration within the range of resonance in such cases presents peculiar features. It appeared of interest to investigate whether such special features could be observed also in the case of the vibrations of a wire produced by the passage of an alternating current through it.

II. Amplitude of the Maintained Vibrations.

To measure the amplitude of the maintained vibrations, a tiny particle of very thin silvered glass was attached to the wire at its middle point, and illuminated. A scale engraved on a mirror was placed behind the wire at right angles to its length, and the drawn-out reflection in the scale was read through a telescope. Two sets of observations were made of the amplitudes of the forced vibrations; in the first set, the length of the wire was kept constant and the tension was varied; and in the second set, the tension was kept constant and the length was varied.

Constant Length and Varying Tension.—A german silver wire of S.W.G. No. 40 was mounted on a sonometer held nearly vertically. A load of 30.9 grammes (which includes twenty half-gramme pieces strung along

[1] Phys. Zeit., 16, pp 137–138, April, 1915.

[2] The effect of the earth's magnetic field, if any, is small, since the experiment succeeds equally well in all positions of the wire.

[3] PHYS. REV., December, 1912, pp. 453.

the length of a silk fiber) was suspended from the free end of the wire. The wire passed over two bridges 50 cm. apart put on the sonometer. The theoretical tension required to make 50 cm. of the wire vibrate with a frequency of 50 per second was equivalent to a load of 25.9 grammes. Starting with a load of 30.9 grammes on the wire, an alternating current of 0.5 amperes, when put on, did not cause the wire to vibrate, but only heated it, the wire slipping over the bridges in consequence of its expansion. This expansion amounted to a few millimeters and persisted as long as the current was on. It did not of course alter the effective length of the wire between the bridges or its tension. The linear density of the wire would, no doubt, have been altered but not to any great extent. The half-gramme masses were then made to drop off one by one, by touching the silk fiber with a glowing match stick. When the tension was thus reduced to 27.4 grammes-weight, the wire started vibrating with an amplitude of 2 mm. With subsequent diminutions of tension, the amplitude increased and attained the value 3.7 mm. when the tension was equal to the calculated value for resonance (25.9 grammes-wt.), and continued steadily to increase as the tension was further reduced until a maximum value of 10 mm. was obtained with a tension of 21.4 grammes-wt. Further reduction of the tension resulted in a sudden and complete cessation of the maintenance. The tension was then increased by the addition of half-gramme loads, but the vibration would not start by itself. The wire was given a small amplitude by plucking, but the vibration died off. But when it was given a sufficiently large amplitude initially, the vibration successfully reëstablished itself, and was maintained with the originally observed amplitude. Similar results were obtained with thicker german silver wires and also with fine copper or nickel wires, and with other current strengths. The maintained amplitudes increased when larger currents were put on, or when thinner wire or wires of greater specific resistance were used.

Constant Tension and Varying Lengths.—Generally similar results were obtained in this case also. The sonometer was placed horizontally and the free end of the wire passed over a pulley, and had a scale pan attached to it. The pan and its contents weighed 25.9 grammes, the load necessary to make the natural period of no. 40 german silver wire of length 50 cm. equal to the period of the alternating current. Two glass prisms were used as the bridges over which the wire passed, the illuminated point being midway between them. With the bridges 47 cm. apart, no appreciable vibration could be noticed when the current was turned on. On increasing the distance between the bridges to 48 cm. a small maintained vibration was observable. On further increasing

the length, even beyond 50 cm., the maintained amplitudes continually increased till a stage was reached at which the amplitude was relatively very large, and any further increase in the distance between the bridges resulted in a sudden and complete collapse of the maintenance. After this stage was passed, shortening the length of the wire a little by bringing the bridges nearer would not restart the vibrations. Small amplitudes when given by plucking died away, but if a sufficiently large initial amplitude were given, the vibration successfully reëstablished itself. Maintenance would start by itself only when the natural period of the wire for small vibrations was less than, equal to, or a very little greater than the period of the current.

When the results of the observations are exhibited graphically we get a figure which is quite unlike the usual type of resonance curve for a vibrator under the action of a periodic force.

As will be seen from Fig. 1, in which the amplitudes for two different current strengths have been plotted against the differences of the squares

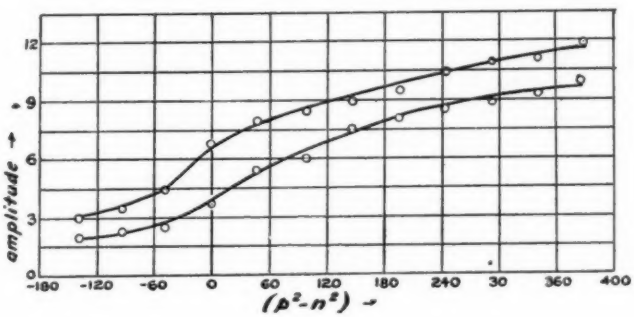


Fig. 1.

of the forced and natural frequencies, we have no "peak" or maximum of resonance, but only a continual increase in the amplitude from one end of the range to the other. This phenomenon finds an explanation in the theory of the maintenance of vibrations by forces of double frequency as developed in the paper[1] by Raman referred to above. The equation for normal vibrations under variable tension is

$$\ddot{w} + k\dot{w} + (n^2 - 2\alpha \sin 2\,pt + \beta w^2)w = 0$$

The solution is

$$w = A_1 \sin pt + B_1 \cos pt + A_3 \sin 3\,pt + B_3 \cos 3\,pt +, \text{ etc.}$$

where A_1, B_1, etc., have to be found by substituting for $\ddot{w}$, $\dot{w}$, and w in

[1] *Note by Editor.*—The reductions suggested require the use of such additional relations as $\sin^3 pt = \frac{3}{4}\sin pt - \frac{1}{4}\sin 3pt$, $2\cos pt \sin 2pt = \sin pt + \sin 3pt$, etc."

the differential equation and equating to zero, the coefficients of sin pt, cos pt, etc. To a first approximation we have

$$\tan E = \frac{B_1}{A_1} = \frac{\alpha - kp}{n^2 - p^2 + F}$$

$$= \left(\frac{\alpha - kp}{\alpha + kp}\right)^{\frac{1}{2}} \text{ where } (n^2 - p^2 + F)^2 = \alpha^2 - k^2p^{2\,2}$$

and

$$F = \frac{3\beta}{4}(A_1^2 + B_1^2).$$

This may be conveniently written in the form

$$\frac{3\beta}{4}(A_1^2 + B_1^2) = (\alpha^2 - k^2p^2)^{\frac{1}{2}} + (p^2 - n^2).$$

Accordingly, when $(A_1^2 + B_1^2)^{\frac{1}{2}}$, which is the amplitude of vibration, is shown graphically against the values of $(p^2 - n^2)$, we should get a parabolic arc, as shown in Fig. 2 for two different values of α.

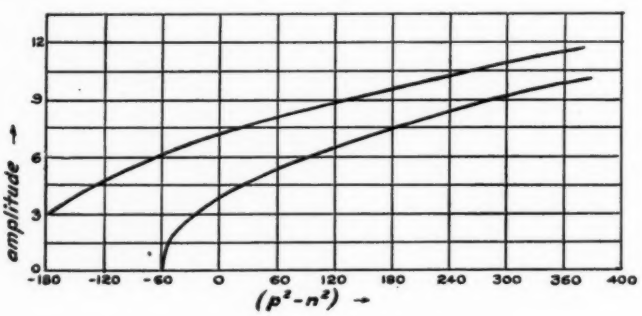


Fig. 2.

It will be seen that the theoretical and the experimental types of resonance curve shown in Fig. 2 and Fig. 1, generally resemble each other except near the extreme end of the range at which very small amplitudes of vibration are obtained. The curves in Fig. 1 show distinct inflections which do not appear in the theoretical curve. The explanation of this small discrepancy will be considered below separately.

III. The Phase of the Maintained Vibrations.

Under the action of periodic forces of the ordinary type, the phase of a maintained vibration alters continually through a series of values of which the extreme difference is 180°, when the natural frequency of the vibration is decreased from a value above to a value below that of the

[1] PHYS. REV., December, 1912, p. 451.

impressed force.[1] The case is entirely different in regard to vibrations maintained by variable spring or tension. According to the formula noted above, the impressed variation of tension being denoted by $-2\alpha \sin 2pt$, the forced vibration would, to a first approximation, be given by

$$w = A_1 \sin pt + B_1 \cos pt$$

where

$$\frac{B_1}{A_1} = \left(\frac{\alpha - kp}{\alpha + kp}\right)^{1/2}.$$

Thus the phase of the maintained vibration would depend only on α and kp, and if these quantities were absolute constants, the phase would remain unaffected when the natural frequency of the wire is altered. In the cases now under consideration, the variation of tension is due to the heating action of the alternating current, and its magnitude and phase for any given current-strength would depend on the experimental conditions. The temperature variations in stationary wires carrying alternating currents have been discussed by Irwin,[2] Drysdale,[3] Ebeling[4] and others, and have been shown to depend on the capacity for heat and the emissivity of the wire. When, as in the present experiments, the wire is in vibration, it would lose heat to the surrounding atmosphere more quickly than usual, and the magnitude and phase of the temperature variation would, no doubt, to some extent depend on the amplitude of vibration. This complicates matters somewhat, and the theoretical conclusion stated above—that the phase of the maintained vibration should be independent of the natural frequency of the wire—may require modification in practice.

In order to study the phase relation of the maintained vibration to the current passing through the wire, an arrangement similar in principle to the vibration microscope was used. The illuminated point of the vibrating wire was observed from a distance through a telescope, the object glass of which was caused to vibrate to and fro in its own plane with the same frequency as the alternating current. This was arranged by fixing the object glass to the middle points of two vertical wires under tension, placed between the poles of a magnet. The alternating current through the sonometer also passed through the two supporting wires in the same direction and caused the vibration of the object glass. The natural frequency of this vibrating system being much smaller than the frequency of the current, the forced vibration of the object glass

[1] Rayleigh's Sound, Vol. 1, p. 48.

[2] Inst. Elect. Engin. Jour., 39, pp. 617–642, Sept., 1907.

[3] Inst. Elect. Engin. Journ., pp. 643–647, Sept., 1907.

[4] Ann.d. Physik., 27, 2, pp. 391–435, Oct., 1908.

may be assumed to have been almost exactly in opposition of phase to the current. The Lissajous figure seen when the illuminated point on the wire is observed through the telescope enables us to infer the phase of its maintained vibration. The observations seem to show that for the current-strengths generally used (0.5 to 0.8 ampere), the phase of the maintained vibration relatively to that of the current is not the same over the whole of the range of resonance, but changes rapidly by about 90° near one end of the range of resonance where the amplitude is very small. Subsequently, over the remaining and much the greater part of the range, the phase remains roughly constant. The general result of the investigation is thus to confirm the indication of the theory of maintenance by forces of double frequency except in the part of the range where the maintained amplitudes are very small.

As regards the origin of the discrepancies between theory and experiment over part of the range of resonance referred to above, it is difficult to pronounce definitely at present. Possibly they are due to a small forced vibration of the ordinary kind produced by the action of the earth's magnetic field on the wire traversed by the alternating current, and superposed on the much larger vibration due to the heating action of the current. Experiments are now in progress to clear up this point and also to study the cases in which the wire is maintained in vibration under the joint action of the electric heating and of external magnetic fields.

GOVERNMENT COLLEGE,
RANGOON, BURMA,
May 8, 1919.

ON THE DIFFRACTION THEORY OF MICROSCOPIC VISION.

By Phanindra Nath Ghosh.

Synopsis.

The changes that take place in the resultant diffraction pattern when a black crossed-grating on a white ground is viewed through a very long rectilinear slit, the axis of which is inclined at an angle of 45° to the lines of the grating and when the slit width is gradually decreased, are described, and reproductions of photographs of typical stages in the image distortion are given. It is shown that when the slit is sufficiently narrow the diffraction field consists of a series of equidistant bright and dark lines at right angles to the chief axis of the slit.

The elemental case of a single black 90° cross on a white ground, when viewed through a slit which is parallel to the bisector of a pair of vertical angles of the cross, is subjected to approximate mathematical analysis, and the contour lines are given both numerically and graphically. It is explained how a synthesis of the elemental pattern accounts, qualitatively at least, for the more general case.

I. Introduction.

According to Abbe's well-known theory, the condition for the optical image of an illuminated structure being an accurate representation of it is that the aperture of the system should be sufficient to permit the entry into it of the entire diffracted streams of light issuing from the structure. The image differs from the object if only a part of the diffracted streams enter the optical system, and the structure would be altogether unrepresented in the image if the aperture be insufficient to permit the entry of the diffracted streams. The mathematical development of the theory for the case of an ordinary grating has been worked out by Lord Rayleigh,[1] Porter,[2] and Wolfke.[3] There are important differences observed between the cases in which the structure is regarded as self-luminous and those in which it is illuminated by an external source, and these have been considered in detail by Mandelstam[4] and also by Lummer and Reiche.[5] The subject has also been extensively studied by Gordon.[6] A case that is of special interest and has often

[1] Rayleigh, Scientific Papers, Vol. IV., p. 243.
[2] Porter, Phil. Mag., January, 1906.
[3] Wolfke, Annalen, 34, 1911; 39, 1912; 40, 1912.
[4] Mandelstam, Annalen 35, 1911.
[5] Lummer and Reiche, Annalen, 35, 1911.
[6] Gordon, Photographic Journal, Dec., 1914.

been used in the illustration of Abbe's theory[1] is that in which the structure is a crossed grating and is viewed through a narrow rectilinear slit. In certain cases, that is, when the slit is held inclined to both sets of lines of the grating, the actual structure of the grating disappears, and is replaced by a spurious set of lines running transverse to the direction of the observing slit. It is proposed in the present paper to consider more fully than has been done by previous writers, the theory of the case of the crossed grating and the manner in which the spurious representation arises, especially the successive stages of the phenomena noticed as the obliquely inclined slit is gradually narrowed down.

It facilitates an understanding of the case of the crossed grating if we consider the phenomena observed when a single dark cross on an uniformly bright ground is viewed through a rectilinear slit. When a sufficiently wide slit is used, the image of the cross is a faithful representation of the object. But as the slit is gradually narrowed down, keeping it parallel to one of the arms of the cross, the line to which the slit is parallel gets fainter and fainter, its apparent width increases, till when the slit has been sufficiently narrowed down, the image disappears, leaving only the other arm of the cross visible in the field. When the

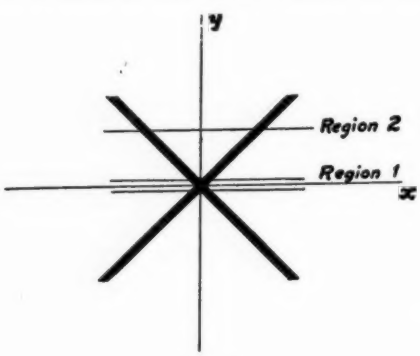
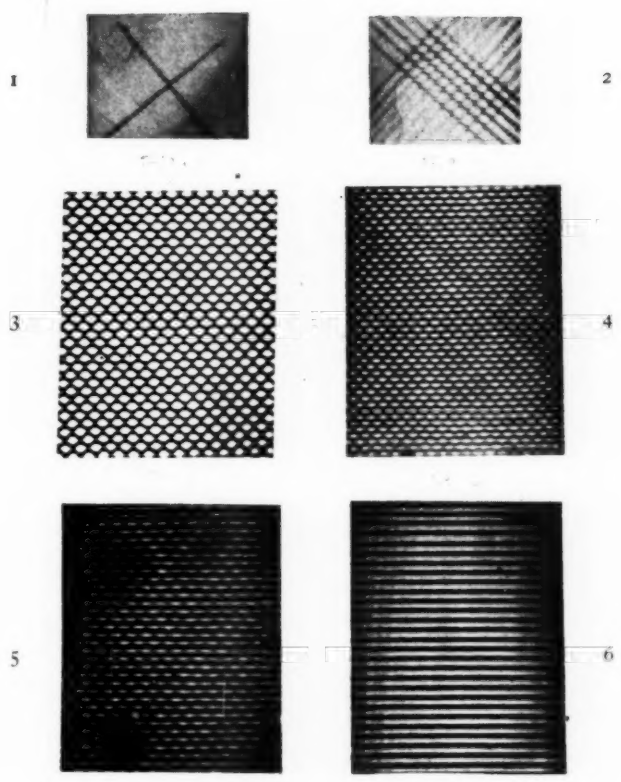


Fig. 7.

lines of the cross are inclined to the slit, both the arms in the image turn grey, their apparent width increases and the junction of the cross undergoes a curious modification. A clean cut bright line which at first is very narrow appears at the junction of the cross in a direction perpendicular to the observing slit, while above and below the junction two dark lines appear. This is shown in Fig. 1. The phenomenon is most clearly seen when the arms make an angle of 45° with the slit.

When a crossed grating made up with alternate dark and bright bars is used as the object, and the slit placed in front of the lens is inclined at

[1] See for instance a paper by Winkelmann, Annalen, 19, 1906.



PHANINDRA NATH GHOSH.

an angle of 45° to the lines of the cross and gradually narrowed down, we have at each point of the crossing of the lines the phenomena described above for a single cross, Fig. 2. In the initial stages, the image of the crossed grating appears as a series of zigzag or wavy lines running transverse to the direction of the observing slit. These stages are clearly shown in Figs. 3 and 4. With the widening of the gaps at the junction

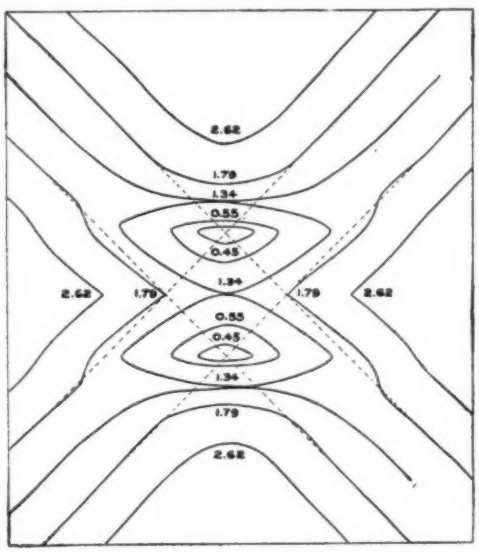


Fig. 8.

of the crosses, the wavy lines straighten up (Fig. 5), and finally we have an entirely spurious set of lines (Fig. 6), running transverse to the observing slit, that is, at an angle of 45° to the two sets of lines of the original grating.

2. MATHEMATICAL THEORY.

In a supplementary paper on the theory of microscopic vision, Lord Rayleigh[1] has given the mathematical theory of the case of a single dark bar on a bright field viewed through a rectilinear slit. We shall here extend the treatment given by Lord Rayleigh to find the contour lines of equal illumination in the image of a dark cross on a bright field formed by a slit held at an angle of 45° to the arms of the cross. In what follows, it will be assumed that the different parts of the luminous ground have no phase relation with each other. To each point on the ground there corresponds a diffraction pattern in the image plane, formed on the line transverse to the direction of the slit containing the geometrical image of the point. The rectilinear slit is assumed to be of great length, so that

[1] Rayleigh, Scientific Papers, V., pp. 118–125.

we can consider independently the diffraction pattern due to each of the strips transverse to the slit into which the luminous ground may be divided.

The diffraction pattern of a radiant point due to a rectilinear slit of width a is a series of points lying on a line transverse to the direction of the slit. The expression for intensity is of the usual form,

$$I = \alpha^2 \frac{\sin^2 \frac{\pi x \alpha}{\lambda f}}{\left(\frac{\pi x \alpha}{\lambda f}\right)^2}$$

or, omitting constants, $\frac{\sin^2 u}{u^2}$.

where f is the focal length of the lens used, and λ the wave-length of the light employed. $x = 0$ is the origin of coördinates.

We shall consider separately the region in which the arms of the cross are superposed and that in which they are separate (shown as regions *I* and *II* in Fig. 7).

Region I.—Considering this region, if the width of the geometrical image of the cross along the line $y = 0$ be 2α, we find that the width gradually increases to 4α at the line $y = \alpha$. Considering the illumination along any strip in this region, and denoting by 2β the width of the geometrical image of the cross along this strip, we get for regions within the geometrical image,

$$I(u) = \int_{-\infty}^{+\infty} \frac{\sin^2 u}{u^2} du - \int_0^{\beta-u} \frac{\sin^2 u}{u^2} du - \int_0^{\beta+u} \frac{\sin^2 u}{u^2} du. \qquad (1)$$

For any point beyond the geometrical image—

$$I(u) = \int_{-\infty}^{+\infty} \frac{\sin^2 u}{u^2} du + \int_0^{u-\beta} \frac{\sin^2 u}{u^2} du - \int_0^{u+\beta} \frac{\sin^2 u}{u^2} du. \qquad (2)$$

Region II.—Here we have to consider the effect of two dark lines each of width 2α situated along the strip at a distance v from the center of symmetry.

Within the geometrical image of the arms of the cross, the expression for the intensity is,

$$I(u) = \int_{-\infty}^{+\infty} \frac{\sin^2 u}{u^2} du - \int_0^{\alpha-u+v} \frac{\sin^2 u}{u^2} du - \int_0^{\alpha+u-v} \frac{\sin^2 u}{u^2} du + \int_0^{u+v-\alpha} \frac{\sin^2 u}{u^2} du + \int_0^{u+v+\alpha} \frac{\sin^2 u}{u^2} du. \qquad (3)$$

In the space outside the arms of the cross,

$$I(u) = \int_{-\infty}^{+\infty} \frac{\sin^2 u}{u^2} du + \int_0^{u-v-\alpha} \frac{\sin^2 u}{u^2} du - \int_0^{u-v+\alpha} \frac{\sin^2 u}{u^2} du + \int_0^{u+v-\alpha} \frac{\sin^2 u}{u^2} du - \int_0^{u+v+\alpha} \frac{\sin^2 u}{u^2} du. \tag{4}$$

In the space between the arms of the cross,

$$I(u) = \int_{-\infty}^{+\infty} \frac{\sin^2 u}{u^2} du + \int_0^{v-u-\alpha} \frac{\sin^2 u}{u^2} du - \int_0^{v-u+\alpha} \frac{\sin^2 u}{u^2} du + \int_0^{u+v-\alpha} \frac{\sin^2 u}{u^2} du - \int_0^{u+v+\alpha} \frac{\sin^2 u}{u^2} du. \tag{5}$$

The integrals can be made to depend upon the sine integral tabulated by Glaisher in the Phil. Trans. for 1870, knowing that

$$\int_{\pm\infty}^{+\infty} \frac{\sin^2 u}{u^2} du = \pi,$$

and that

$$\int_0^x \frac{\sin^2 u}{u^2} du = \int_0^{2x} \frac{\sin u}{u} du - \frac{\sin^2 x}{x} = Si2x - \frac{\sin^2 x}{x}.$$

We have

$$(1) = \pi - Si2(\beta - u) + \frac{\sin^2(\beta - u)}{\beta - u} - Si2(\beta + u) + \frac{\sin^2(\beta + u)}{\beta + u},$$

$$(2) = \pi + Si2(u - \beta) - \frac{\sin^2(u - \beta)}{u - \beta} - Si2(\beta + u) + \frac{\sin^2(\beta + u)}{\beta + u},$$

$$(3) = \pi - Si2(\alpha - u + v) - Si2(\alpha + u - v) + Si2(u + v - \alpha) - Si2(u + v + \alpha) + \frac{\sin^2(\alpha - u + v)}{\alpha - u + v} + \frac{\sin^2(\alpha + u - v)}{\alpha + u - v} - \frac{\sin^2(u + v - \alpha)}{u + v - \alpha} + \frac{\sin^2(u + v + \alpha)}{u + v + \alpha},$$

$$(4) = \pi + Si2(u - v - \alpha) - Si2(u - v + \alpha) + Si2(u + v - \alpha) - Si2(u + v + \alpha) - \frac{\sin^2(u - v - \alpha)}{u - v - \alpha} + \frac{\sin^2(u - v + \alpha)}{u - v + \alpha} - \frac{\sin^2(u + v - \alpha)}{u + v - \alpha} + \frac{\sin^2(u + v + \alpha)}{u + v + \alpha},$$

$$(5) = \pi + Si2(v - u - \alpha) - Si2(v - u + \alpha) + Si2(u + v - \alpha) - Si2(u + v + \alpha) - \frac{\sin^2(v - u - \alpha)}{v - u - \alpha} + \frac{\sin^2(v - u + \alpha)}{v - u + \alpha} - \frac{\sin^2(u + v - \alpha)}{u + v - \alpha} + \frac{\sin^2(u + v + \alpha)}{u + v + \alpha}.$$

Using these formulæ, the intensities at various points in the field have been computed and are shown below.

Table of Intensities for Slit Width $a = \lambda f/\pi$.

y.	x=0.	x=.5.	x=1.	x=1.5.	x=2.	x=2.5.	x=3.
0	1.342	1.53	1.797	2.25	2.62	2.89	3.06
.2	1.087	1.27	1.564	2.01	2.46	2.77	3.006
.5	.77	.95	1.25	1.71	2.21	2.58	2.88
.8	.55	.72	.96	1.41	1.808	2.37	2.72
1.0	.45	.57	.83	1.19	1.71	2.25	2.56
1.2	.878	.87	.96	1.22	1.61	2.08	2.49
1.5	1.358	1.297	1.25	1.47	1.65	1.79	2.25
2.0	2.102	1.88	1.71	1.52	1.34	1.59	1.79
2.5	2.608	2.53	2.24	1.794	1.49	1.34	1.49
3.0	2.968	2.88	2.61	2.25	1.794	1.49	1.34

The contour lines of equal illumination have been drawn from the above data (Fig. 8); they clearly show the oval shaped contours of the dark areas above and below the junction of the cross,and also the increased illumination transverse to the rectilinear slit at the junction of the cross. The crossed grating can be considered as an assemblage of a large number of dark crosses with their junctions lying along a system of lines parallel to the x axis. The form of the lines of equal illumination in then neighbourhood of each cross would be influenced to some extent by neighbouring crosses. At first, the oval-shaped darkspots at the junctions touching each other would form a zigzag boundary running transverse to the slit, and when the slit-width has been sufficiently narrowed down, these zigzag boundaries would be drawn out into continuous lines with bright spaces between them. Thus, finally, in the image, instead of the original crosses, we get the spurious representation of a set of lines transverse to the slit.

In conclusion, I beg to record my best thanks to Professor C. V. Raman for his help and encouragement during the course of the investigation.

UNIVERSITY COLLEGE OF SCIENCE,
CALCUTTA, INDIA,
May 7, 1919.

RESONANCE AND IONIZATION POTENTIALS FOR ELECTRONS IN THE MONATOMIC GASES ARGON, NEON AND HELIUM.

BY H. C. RENTSCHLER.

SYNOPSIS.

The monatomic gases argon, neon and helium were tested for resonance potential. The method used was that first described by Tate in his work with mercury vapor. As a further check of the results a method similar to Lenard's method was used.

Experiments are described which show the necessity of properly freeing the metal electrodes from adsorbed and occluded gases.

The curves obtained indicate that argon has a resonance potential and that neon and helium do not.

INTRODUCTION.

TWO types of inelastic impacts between electrons and atoms of a number of metallic vapors[1] have been found to exist. The first type occurs when the colliding electron displaces an electron of the atom without removing it from the atom. The potential through which the colliding electron must fall to acquire the necessary energy to produce this displacement is known as the resonance potential for the vapor in question. The second type of inelastic impact occurs when the colliding electron removes an electron from the atom and produces ionization. The present investigation was undertaken to test the gases argon, neon and helium for these two types of inelastic impact.

APPARATUS AND METHOD.

The arrangement of the apparatus is shown in Fig. 1. A tungsten filament f was heated to incandescence by the battery B. Surrounding this filament and concentric with it was a nickel net n of about 100-mesh to the inch made from 4-mil wire. The diameter of the net was about 1.5 cm. and surrounding this in turn was a nickel cylinder C about 2 cm. in diameter.

[1] Mercury: Tate, PHYS. REV., 7, p. 686, 1916. Goucher, PHYS. REV., 8, p. 561, 1916. Davis and Goucher, PHYS. REV., 10, p. 101, 1917.

Sodium: Tate and Foote, J. Wash. Acad. Sci., 7, p. 517, 1917.

Cadmium: Tate and Foote, Bur. Standards Sci. Paper No. 317.

Sodium, Cadmium, Potassium and Zinc: Tate and Foote, Phil. Mag., 36, p. 64, 1918.

Magnesium and Thallium: Foote and Mohler, Phil. Mag., 37, p. 33, 1919.

Arsenic, Rubidium, Cæsium: Foote, Rognley and Mohler, PHYS. REV., 13, p. 59, 1919.

Before assembling the tube as shown in Fig. 1, the net and cylinder were freed from occluded and adsorbed gases. This was done by sealing them in separate tubes with tungsten filaments along their axes. These tubes were exhausted with a diffusion pump and the net and cylinder bombarded by thermions from their respective filaments until no more gases could be liberated from them even when the bombardment was so violent that they were heated to as near their melting point as was safe. After allowing them to cool in the vacuum, the net and cylinder were taken out of the respective bulbs and mounted in the test bulb as shown in Fig. 1. The test bulb was exhausted by a diffusion pump and heated to about 400° C. to remove gases from the walls of the glass. The net and cylinder were again bombarded by thermions from the filament to remove gases adsorbed while transferring them to the experimental bulb. It is desirable to have a short filament in the experimental tube so as to have as small a potential drop along the filament as possible. With a short filament it would have been difficult to get sufficient bombardment of the cylinder through the net to heat it hot enough to drive out the occluded gases and hence the pre-treatment given the net and cylinder.

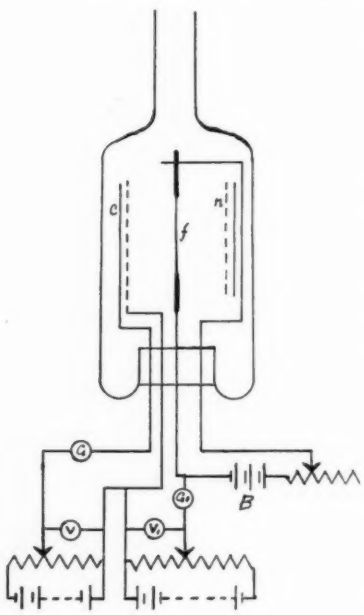


Fig. 1.

In treating nickel to remove the occluded gases a rather interesting phenomenon was noticed. From the length and diameter of the filament the electron current from the filament to the cylinder was calculated. At first the current that could be obtained was only about one seventh of the calculated current. After the bombardment had proceeded for about ten minutes the current gradually increased and after the nickel was entirely free from gas the calculated thermionic current was easily obtained. This was probably due to a poorly conducting coating on the surface of the nickel.

The tube was now filled to the desired pressure with the gas to be tested. A trap immersed in liquid air, between the ionization chamber and the pump prevented mercury vapor from the pump and gauge from reaching the ionization chamber. A constant retarding potential

was applied between the net and the outer cylinder and both the current from the hot filament and that part of the current which reached the outer cylinder against the retarding field were measured by the galvanometers G_1 and G, as functions of the accelerating potential between the filament and the net. This was done for different retarding potentials and for different pressures. A decrease in the electron current to the outer cylinder with increase in accelerating potential generally indicates inelastic impact of the electrons while a sudden increase in the total current from the hot filament indicates ionization. A decrease in the cylinder current without a simultaneous increase in the total current from the filament would indicate that the inelastic impact was due to resonance and not ionization.

Second Method.

As a further check on the results, the outside cylinder was maintained at a sufficient negative potential with reference to the filament so that the electrons liberated from the filament were not able to reach the cylinder. This was done either by using a retarding potential always greater than the accelerating potential or by maintaining a constant difference of potential between the filament and outer cylinder (cylinder negative). A current through the galvanometer in the cylinder circuit may be due to positive ions reaching the cylinder or photo-electrons liberated from the cylinder or to a combination of these two effects. Such a current without ionization, indicated by a break in the total current from the filament, must be due to photoelectric action. With the metallic vapors resonance is accompanied by single line radiation. If this radiation is of sufficiently short wave-length to produce photoelectric action this method can be used as a check for either resonance or ionization in as much as ionization is always accompanied by recombination and therefore radiation of the complete spectrum.

Results.

Argon.—The argon used in these tests was purified by circulating the gas through an arc between calcium electrodes until no traces of other gases could be noticed in the spectrum of the argon. Tests were made at various pressures ranging from a fraction of a millimeter of mercury to several millimeters. A few of the curves obtained are shown in Fig. 2.

Curve I shows the variation of the current to the outside cylinder for a retarding potential of 2 volts between the net and cylinder. Curves 2, 3 and 4 are similar curves for retarding potentials of 3, 4 and 6 volts respectively. The first decrease in these curves occurs at about 3 volts

higher than the retarding potentials for the corresponding curve. Thus curve *1* starts to decrease at about 5 volts which is 3 volts higher than the retarding potential for this curve. This decrease is probably produced by a reflection or reëmission of electrons from the adsorbed argon on the surface of the cylinder. Curve *6* is a similar curve obtained when the apparatus was thoroughly evacuated after taking curves *1*

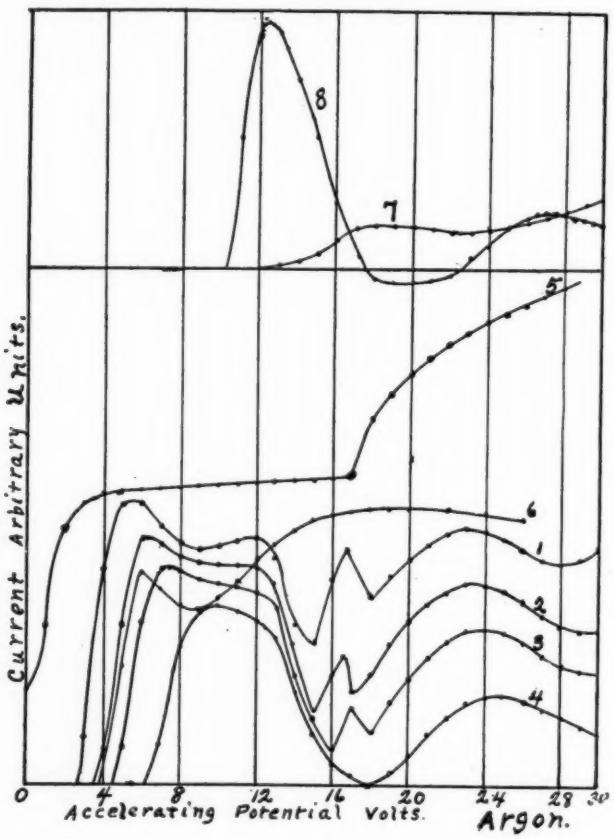


Fig. 2.

to *4*, but before the adsorbed argon was removed from the cylinder. The retarding potential for this curve was 3 volts. After bombarding the cylinder in the high vacuum again the break in curve *6* at 6 volts was no longer obtained, indicating that reflection or reëmission took place from the adsorbed gas on the cylinder.

All the curves *1* to *4* show a sharp decrease in the cylinder current beginning between 12 and 13 volts accelerating potential. The total current curve *5* indicates ionization at about 17 volts. At this same potential there is a sharp break in the curves *1* to *3* as there should be

since the electrons producing ionization collide inelastically. For curve *4* the retarding potential was too high for the current to begin increasing before this decrease at 17 volts occurs. The decrease between 12 and 13 volts indicates resonance. To determine the resonance potential more accurately the effect of reflection of the electrons from the outer cylinder must be eliminated. For curve *8* a retarding potential of 11 volts was therefore used. The inelastic impact due to resonance should show again at twice the resonance potential. Curves *1* to *4* show the effect but the disturbance due to reflected electrons from the cylinder influences the value at which the break in the curve occurs. That is, an electron may lose its energy in producing ionization at 17 volts and then fall through such a potential that upon striking the cylinder it is reflected at a net potential slightly lower than twice the resonance potential. For curve *1* this should occur for an electron velocity corresponding to about 3 volts when striking the cylinder, that is, for an accelerating potential of 17 plus 3 plus 2 (the retarding potential) or about 22 volts. For curves *2*, *3* and *4* the reflection effects should enter at about 23, 24 and 26 volts respectively. The second resonance potential for each of these curves should occur at twice the resonance potential. These two effects overlap and make it impossible to determine the second resonance points from curves *1* to *4*. For curve *8* this disturbance due to reflection or reëmission does not complicate the curve but the change in curvature is so gradual that it is difficult to estimate the second resonance point accurately. A number of curves similar to *5* and *8* were taken for retarding potentials as given below.

Regarding Potential.	First Resonance Point.	Ionizing Potential.
11	12.2	17
10.6	12	17.2
11.5	12.5	17
10	12	17
10.5	12.5	16.8
11	12.6	17

The average value for the resonance potential is thus 12.3 volts and the ionizing potential 17 volts. The second peak for the six curves similar to 8 at about 26.5 volts all occur at a potential varying not more than half a volt from the average.

To check the results thus obtained method 2 described above was used. The outer cylinder was maintained negative by a few volts with reference to the negative end of the filament while the potential of the net was varied. The current through the galvanometer connected to the outer cylinder as a function of the net potential is represented by curve *7*.

A positive ordinate for curve 7 represents an electron stream away from the cylinder or positive ions to the cylinder. For the other curves the opposite direction is taken. A curve showing the total current from the filament in terms of the accelerating potential was taken simultaneously with curve 7. This curve is exactly similar to curve 5 showing ionization at 17 volts. The current of curve 7 starting between 12 and 13 volts is therefore clearly due to photoelectric action on the outer cylinder.

Discussion of Results for Argon.—The only previous determination of the ionizing potential of argon was made by Franck and Hertz.[1] The value given by them is 12 volts not correcting for the initial velocity of emission of the electrons. The present investigation indicates that they were probably dealing with the resonance potential and not the ionizing potential. The method used by them (Lenard's method) does not distinguish between the two. Recently Lyman[2] has identified lines as short as 833 A.U. with argon. From the quantum theory $h\nu = eV$ this would require an ionizing potential of about 15 volts to produce this line as the shortest line of the spectrum. In his experiments Lyman had to use long paths for his light to travel and no doubt absorption produced a decided disturbance and it is probable that there are lines in the argon spectrum shorter than he was able to detect with his apparatus. The shortest helium line he could detect has a wave-length of about 600 A.U. while Richardson[3] and Bazzoni by a photoelectric method find this limit to be between 420 and 470 A.U. The value of the ionizing potential determined by the present experiment indicates the shortest argon line to have a wave-length of about 750 A.U. calculated from the quantum relation $h\nu = eV$.

Neon.—The neon used for these tests was originally prepared by Claude. Last traces of helium were removed by fractionation from cocoanut charcoal cooled in liquid air. The purification was continued until no trace of any other gases could be detected in the spectrum. Different retarding potentials varying from 1 to 18 volts between the net and cylinder were used for pressures varying from a tenth of a millimeter of mercury to 5.9 millimeters. The best results were obtained for the higher pressures. In Fig. 3 a few of the curves obtained for neon are shown. The pressure in this case was 5.9 mm. Curve 1 represents the total electron current from the filament as a function of the accelerating potential between the filament and net. This curve indicates ionization at about 19.5 volts. Curve 2 represents the electron current which reaches the outer

[1] Franck and Hertz, Verh. d. Deutsch. Phys. Ges., 15, p. 34, 1913.

[2] Lyman, Ast. Phys. Jour., 43, p. 89, 1916.

[3] Richardson and Bozzoni, Phil. Mag., 6–34, p. 285, 1917.

cylinder. This current begins for a net potential of about 15.5 volts and increases as the net potential increases until ionization sets in when the current reverses direction. This is due partly to positive ions reaching the cylinder and partly to photoelectric action on the cylinder. This reverse current diminishes continuously as the net potential is raised to about 33 volts. This may be brought about by the combination of several causes—first with increase of net potential the ionization takes place nearer the filament diminishing chances of positive ions reaching the

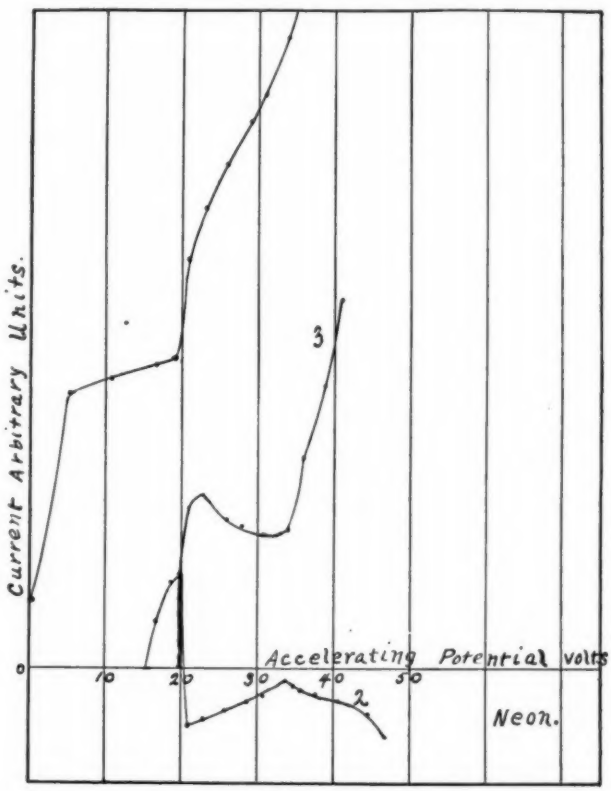


Fig. 3.

cylinder. In the second place recombination and therefore radiation thus produced takes place further from the cylinder accompanied by greater absorption of the radiation before it reaches the cylinder and also by increased shielding of the cylinder by the net and hence less photoelectric action on the cylinder. Third, as the net potential is raised a value is reached when an electron once having suffered inelastic collision in producing ionization will again have acquired sufficient energy to

reach the cylinder thus diminishing the apparent reverse current. Curve *3* was obtained with the cylinder negative with reference to the filament (method *2*). At the potential of the net at which curve *1* indicates ionization curve *3* shows positive ions reaching the cylinder or photo-electrons leaving it or both. This current reaches a maximum and then diminishes to about 33-volt net potential. The cause for this decrease is probably the same as the first and second explanations given for curve *2*. At about 33 volts there is a break in curves *2* and *3*. This break is noticeable in curve *1* but not as definitely marked in *2* and *3*. These breaks indicate second ionization. Normally this second ionization should occur at twice the voltage at which the first occurs whereas in this case it is about 6 volts less than twice the value. Attempts were made to get a third and fourth break but when the net potential was raised to about 40 volts the cylinder current never reached a steady value for a definite net potential but continued increasing. While this drift of the galvanometer was taking place the filament appeared to be getting noticeably hotter probably due to bombardment of the filament by positive ions thus approaching something similar to arc conditions. This general effect is probably present to some extent as soon as the ionization potential is exceeded and may be a partial explanation for the second ionization point being less than twice the potential of the first.

The results obtained with neon show no indication of a resonance potential. The indications are that the ionizing potential is about 19.5 volts, not allowing for the initial speed with which the electrons leave the filament. The usual way of determining this speed is to take the difference between the potential at which first ionization occurs and that at which the same electron produces ionization twice as true ionizing potential and subtracting from this the potential for single ionization. As pointed out before, this is not practical in the present case.

The only other determination of the ionizing potential of neon is that made by Franck and Hertz using Lenard's method. The value given by them is 16 volts.

Helium.—The helium used was obtained from monazite sand and Thorianite. The gas was purified by passing it through a tube filled with cocoanut charcoal immersed in liquid air. The curves of Fig. 4 show the variation of the electron current from a filament to the concentric cylinder without a net as a function of the accelerating potential from the filament to the cylinder.

For curve *1* the cylinder was nickel. The ionization chamber was baked out but the cylinder was not specially treated by bombardment to remove occluded gases and impurities. After taking a number of curves

similar to curve *1* the helium was pumped out and the cylinder heated by making the cylinder positive with reference to the hot filament and thus bombarding it by thermions until no more gas could be removed. After this treatment the curve obtained for the same pressure is represented by *2*. Curve *3* shows similar result obtained with a treated copper cylinder.

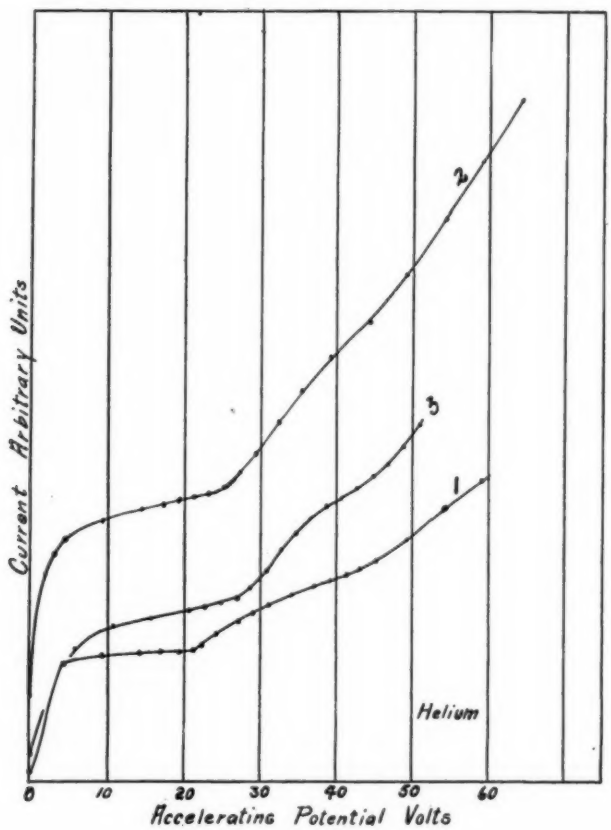


Fig. 4.

To determine whether helium has a resonance potential, the same methods were used as for argon and neon. Tests were made for pressures varying from a fraction of a millimeter to 1.6 centimeters of mercury and for retarding potentials between the net and cylinder varying from zero volts to 32 volts. In some cases an accelerating potential was even used. A few of the curves obtained are shown in Fig. 5.

Curve *1* shows the electron current to the outer cylinder for 1.5 volts retarding potential, curve *2* for zero volts retarding potential and curve *3* for an accelerating potential of 3 volts. The pressure in this case was

3 millimeters of mercury. The net and cylinder were both nickel. Every curve taken with a nickel net and a nickel cylinder shows a sharp increase in the electron current to the outer cylinder starting between 20 and 25 volts, rising to a maximum value and suddenly dropping the instant ionization as indicated by corresponding curve *4* starts. This increase is repeated again between 40 and 50 volts net potential. Curve

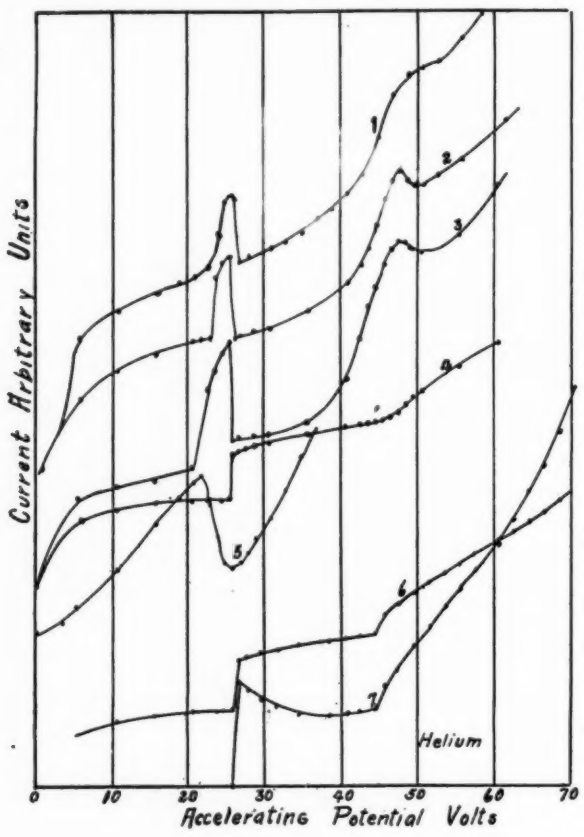


Fig. 5.

5 is a similar curve using a copper net and a copper cylinder for 2-volt retarding potential and a pressure of about 4 millimeters of mercury. The copper net had a mesh of about 60 to the inch as against 100 for the case of nickel. Otherwise the tests using copper elements were exactly similar to those with the nickel net and cylinder. With a copper net and a nickel cylinder curves similar to curve *5* were obtained. It is difficult to see how the difference in the mesh of the net could produce as much difference in the character of the curves as there is between curves *1* and *5*. Again the sharp increases in the current of curves *1*, *2*,

etc., starting around 21 volts are equally difficult to explain. Clearly the increase cannot be due to ionization in the space between the net and the outer cylinder starting before the ionization starts inside of the net. This might explain curve *3* but cannot explain *1* and *2* and a number of similar curves taken for retarding potentials up to 10 volts or more—for if ionization took place in this space positive ions would be driven to the outer cylinder and thus produce a smaller effective electron current to the cylinder instead of an increased current. Again it is hardly possible that the increase could be produced by reflection or reëmission of electrons from the net after a certain electron velocity is reached. This might explain curve *1* and similar curves taken with a retarding potential between the net and cylinder, but does not explain curves *2* and *3* because reflection should take place from the cylinder as well as from the net for the cases of *2* and *3*.

Dr. F. Horton[1] and Ann C. Davis in a recent note in Nature state that they find a resonance potential of 20.4 volts and an ionizing potential of 25.6 volts for helium. In order to explain these current increases on the basis of resonance accompanied by photoelectric action on the net and a consequent increase in the number of electrons to the outer cylinder, several assumptions are necessary. First the radiation due to resonance must be strongly absorbed before it reaches the outer cylinder, otherwise this photoelectric action on the cylinder would tend to neutralize the effect of the action on the net. In the second place the electron producing resonance will not have sufficient velocity left to reach the cylinder against a retarding field. The photoelectric action on the net must therefore liberate several electrons for each electron producing resonance in order to explain the increased current effect. That is to say an electron having a velocity corresponding to 20.4 volts must collide inelastically and produce resonance and consequently radiation. The original electron loses its energy and will not reach the cylinder against the retarding field. This radiation falling on the net must liberate several electrons of sufficient velocity to reach the cylinder against the retarding field in order to account for this sharp increase in current to outer cylinder. Considering that not nearly all the radiant energy can fall on the net and that the sharp increase in current is present even when using as much as 10 volts retarding potential this explanation seems out of the question.

Curve 4 represents the total current from the filament to the net and cylinder as a function of the accelerating potential, taken simultaneously with curve *1*. Curve *6* is another total current curve taken

[1] Horton and Ann C. Davis, Nature, Feb. 13, 1919.

simultaneously with curve 7. Curve 7 represents an electron current away from the outer cylinder or a positive current to the cylinder when the cylinder was maintained negative by 10 volts with reference to the negative end of the filament. (Method 2.) This current is zero until ionization (shown by the sudden break in curve 6) starts. This ionization is accompanied by recombination and hence radiation so that some positive ions would be expected to reach the outer cylinder and photoelectrons leave it. For exactly the same reason as in the case of neon, this curve should reach a maximum, then decrease and show a sudden break again when the same electron produces ionization for the second time. This second break as in the case of neon occurs at less than twice the potential at which the first ionization occurs, and the same explanation applies to this as was given for neon. Attempts were made to get a third break in the different curves but as in the case of neon there was a gradual increase in current for a constant accelerating potential at the higher net voltages, which made results meaningless.

The results with helium do not indicate a resonance potential. The sharp increases in the electron current to the outer cylinder are probably due to some unexplained action of the net.

In order to get the exact value of the ionizing potential for helium from these curves several corrections are necessary. First the initial velocity with which the electrons leave the filament must be known. As was pointed out before, this initial velocity cannot be determined in the usual manner and therefore cannot be corrected for. In the second place K. T. Compton[1] and Benade point out that the loss of energy of an electron when it strikes a molecule of helium is not entirely negligible. With the gas under the pressures used in this experiment the average number of collisions of an electron with different molecules before it reaches the net is large and the correction factor due to these collisions may be quite appreciable. This correction could be calculated using Compton and Benede's results if the effect of space charge, etc., upon the electric field between the filament and net are taken into consideration. No attempt was made to do this since other correction factors could not be definitely determined.

Taking the breaks in the various curves to determine the ionizing potentials the values for pressures varying from about 1 millimeter to 1.6 centimeters do not differ by more than 1.5 volts, from the average value of 26 volts obtained from some sixty curves.

[1] K. T. Compton and Benade, PHYS. REV., 11, p. 184, 1918.

Conclusions.

The results of these tests indicate that argon has a resonance potential of about 12 volts and an ionizing potential of about 17 volts. Neon and helium do not show resonance. The ionizing potentials for neon and helium are higher than those obtained by other observers. The main purpose of these experiments was rather to test for resonance and not to determine the ionizing potentials with great accuracy.

Nutting[1] and Tugman point out that helium persistently gives the same spectrum and that no lines appear or disappear as the excitation is varied. The same appears to be true for neon while the character of the argon spectrum and of the metallic vapors is very materially changed by changing the method of excitation. While this is no direct proof for a resonance potential for argon and not for neon and helium it may perhaps be considered a further verification of the results of this experiment.

[1] Nutting and Tugman, Bulletin Bureau of Standards, Vol. 7, No. 1, 1910.

ON THE X-RAY ABSORPTION FREQUENCIES CHARACTERISTIC OF THE CHEMICAL ELEMENTS.

BY WILLIAM DUANE AND KANG-FUH-HU.

SYNOPSIS.

Object.—The object of this paper is to record in some detail the data presented to the American Physical Society at its meeting on April 27, 1918.[1]

Blake and Duane[2] had determined the critical absorption wave-lengths associated with the K series of x-rays for most of the chemical elements from bromine (atomic number 35) to cerium (atomic number 58) by measuring the currents in an ionization chamber attached to an x-ray spectrometer. The research reported in this paper extends the measurements to chemical elements of lower atomic number, as far as manganese (atomic number 25). The same apparatus was used as in the previous research, except that the x-ray tube employed had a long glass tube attached to it carrying a thin glass window at its end. This window lay close to the spectrometer slit, and the device materially reduced the absorption of the long x-rays by the air and glass.

Importance in Theory.—These critical absorption wave-lengths have considerable importance in connection with the theories of the structure of matter and the mechanism of radiation, for they represent frequencies of vibration that are the highest x-ray frequencies definitely known to be characteristic of the chemical elements.

Results.—From the experiments it appears that the square root of the critical absorpion frequency is not quite a linear function of the atomic number.

If we calculate the velocity v of an electron in the x-ray tube required to produce the radiation from the equation

$$\frac{1}{2}\frac{m_0 v^2}{\sqrt{1-\frac{v^2}{c^2}}} = h\nu \qquad (1)$$

using for ν the critical absorption frequency, we find that v is a linear function of N, namely

$$v = v_0 (N - 3/2),\ v_0 = .00678xc \qquad (2)$$

for all the chemical elements from manganese to cerium.

Further equation (2) gives the critical velocity for the chemical elements as far as magnesium, if we use data obtained from emission spectra.

It will be noticed that equation (1) contains the expression for the *transverse* mass of the electron.

IN Moseley's classical experiments[3] on x-rays he showed that the square roots of the frequencies, ν, of corresponding lines in the characteristic emission spectra of the chemical elements are very nearly linear functions of the atomic number, N, of those elements. Recent researches have confirmed and extended these results. The wave-lengths of about fifteen

[1] PHYS. REV., June, 1918, p. 488.

[2] PHYS. REV., Dec., 1917, 697.

[3] Phil. Mag., April, 1914, p. 703.

lines in each of the emission spectra of a large number of chemical elements have been measured and tabulated. The graphs[1] representing $\sqrt{\nu}$ for corresponding lines in these spectra as functions of N are not quite straight. They bend slightly upward, the curvature becoming more marked as the frequency increases.

Attempts have been made to deduce empirical formulas for ν in terms of N, and some of these,[2] with four arbitrary constants, seem to fit the data very well.

The equations[3] in Bohr's theory of radiation contain the mass m of the electron. Since m varies with the electron's velocity the square roots of these expressions for the frequencies of rotation and vibration are not quite linear functions of N. The deviation from the straight line law is much the same as that which appears in the graphs representing experimental data.

It might be expected that the critical absorption frequencies associated with the emission series of x-rays would bear the most fundamental and, perhaps, the simplest relations to the atomic numbers; for they appear to be the most important frequencies characteristic of the chemical elements. Their importance rests upon the following facts. (*a*) A critical absorption frequency equals the critical ionization frequency associated with the same x-ray emission series.[4] The latter probably means the critical frequency for the characteristic emission of electrons with definite energy from the atom. (*b*) The difference between two critical absorption frequencies equals the frequency of one of the emission lines characteristic of the chemical element.[5] (*c*) A critical absorption frequency, substituted in the quantum equation, $Ve = h\nu$, gives the voltage V required to produce the emission series associated with it.[6] (*d*) A K critical absorption frequency is the highest x-ray frequency known to be characteristic of its chemical element. It lies very close to, but slightly above, the highest frequency in the corresponding emission series.[7]

De Broglie has made a series of interesting and important experiments, in which he measured the critical absorption wave-lengths characteristic of a large number of chemical elements. He used an x-ray spectrometer with a rock salt crystal, and determined the position of the beam of rays reflected from the crystal by means of a photographic plate.

[1] Friman, Phil. Mag., Nov., 1916, p. 497.
[2] Uhler, PHYS. REV., April, 1917, p. 325.
[3] Phil. Mag., September, 1913, p. 476.
[4] Duane and Hu, PHYS. REV., June, 1918, p. 489; Dec., 1919.
[5] Duane and Shimizu, PHYS. REV., April, 1919, p. 306; July, 1919.
[6] Webster, PHYS. REV., June, 1916, p. 599; and Duane and Hu, *l.c.*
[7] Duane and Hu, *l.c.*

Owing to the difficulty of estimating by this method the effect due to the penetration of the x-ray into the reflecting crystal, and that due to the widths of the spectrometer's slits, Prof. F. C. Blake and one of us[1] decided to undertake a series of measurements using the ionization method of detecting the reflected beam. The data obtained in these experiments include the critical absorption wave-lengths associated with the K series of x-rays for all but two of the known chemical elements from bromine ($N = 35$) to cerium ($N = 58$).

The equation $\nu = \nu_0 (N - 3.5)^2$, in which ν_0 is the Rydberg fundamental frequency, namely 109,675 multiplied by the velocity of light, approximately represents the critical absorption frequencies ν corresponding to these wave-lengths. There appears to be, however, a small systematic variation from the law represented by the equation.

The object of the research reported in this paper has been to extend the measurements to chemical elements of lower atomic number than that of bromine, and we have succeeded in obtaining the critical absorption wave-lengths associated with the K series for all the chemical elements from bromine ($N = 35$) to manganese ($N = 25$).

The magnitudes of the frequencies belonging to chemical elements having small values of N give us the best determinations of the quantity k in formulas of the general form

$$\nu = \nu_0(N - k).$$

The x-rays in the K series of chemical elements of low atomic numbers are comparatively long; and, since the coefficient of absorption of x-rays varies approximately as the cube of the wave-length, the absorption of the rays of long wave-length by the glass walls of the x-ray tube itself becomes of great importance. In order to allow as much radiation of long wave-length to emerge from the tube as possible we designed, and had constructed an x-ray bulb with a long glass side tube attached to it. This tube extended out toward the x-ray spectrometer, and carried at its end a thin glass window, which lay close to the spectrometer's slit. The device markedly reduced the absorption of the x-rays by the glass and air.

Except for this side tube attached to the x-ray bulb the apparatus used in the experiments on chemical elements from manganese to bromine was exactly the same as that employed in the experiment on chemical elements from bromine to cerium. A detailed description of this apparatus may be found in the PHYSICAL REVIEW for December, 1917, on page 624.

[1] PHYS. REV., December, 1917, p. 697.

The electric current exciting the x-ray tube came from a high-tension storage battery containing 20,000 cells, and the voltage across the tubes terminals was kept constant by slowly changing a water resistance in series with it. An electrostatic voltmeter, calibrated by means of a measured current from the storage battery flowing through a known metallic resistance of 890,000 ohms, served to measure the voltage. The current through the tube, amounting to several milliamperes was measured by a milliameter. Two slits in lead blocks placed between the x-ray tube and the reflecting crystal defined the beam of rays and the third slit, in front of the ionization chamber, was wide enough to allow all the reflected rays to pass. This arrangement eliminates a correction for the penetration of the rays into the crystal. This correction, however, is very small for x-rays of long wave-lengths.

The absorbing screen consisted of a thin layer of the chemical element, or of one of its salts, and lay between the x-ray tube and the first slit.

We made measurements on both sides of the zero line of the spectrometer, and obtained curves for each chemical element representing the ionization currents as functions of the readings of the verniers attached to the crystal table, similar to the curves shown on page 701 of the PHYSICAL REVIEW for December, 1917. The sharp drops in these curves correspond to the critical absorption wave-lengths, the angular breadths of the drops representing the effects due to the widths of the slits.

To determine the grazing angles of incidence θ to substitute in the formula for the wave-length

$$\lambda = 2a \sin\theta = 6.056 \times \sin\theta \times 10^{-8} \text{ cm.}$$

we measured from the mid-points in the drops. The breadths of the drops amount to a few minutes of arc, and the grazing angles of incidence to several degrees. The positions of the mid-points can be estimated to within 1/10 per cent. of the values of the grazing angles.

The slopes of the curves above and below the drops depend upon the setting of the instrument, the thickness of the absorbing layer, the relation between the voltage applied to the x-ray tube and that required to produce x-rays as short as the critical rays, etc.

The following Table I. contains the critical absorption data for the K series of all the known chemical elements from manganese ($N = 25$) to cerium ($N = 58$), except the two gases xenon and krypton. The critical absorption wave-lengths appear in the third column, and the square roots of the corresponding vibration frequencies, in the fourth. The $\sqrt{\nu}$ cannot quite be represented by a linear function of N. The values of $\sqrt{\nu}$ differ from such a function by several per cent.

TABLE I.[1]

Chemical Element.	Automic Number.	$\lambda \times 10^8$.	$\sqrt{\nu} \times 10^{-9}$.	Ratio $\beta = v/c$.	$\beta_0 \times 1{,}000$.
Cerium	58	.3068	3.127	.3828	6.775
Lanthanum	57	.3188	3.068	.3761	6.778
Barium	56	.3307	3.012	.3698	6.785
Cæsium	55	.3444	2.952	.3629	6.783
Iodine	53	.3737	2.833	.3494	6.784
Tellurium	52	.3896	2.775	.3426	6.783
Antimony	51	.4065	2.717	.3358	6.784
Tin	50	.4242	2.659	.3291	6.786
Indium	49	.4434	2.601	.3223	6.786
Codnium	48	.4632	2.545	.3157	6.790
Silver	47	.4850	2.488	.3089	6.789
Palladium	46	.5075	2.431	.3023	6.794
Rhodium	45	.5330	2.373	.2953	6.790
Ruthenium	44	.5584	2.319	.2889	6.797
Molybdenum	42	.6180	2.203	.2751	6.794
Niobium	41	.6503	2.149	.2685	6.797
Zirconium	40	.6872	2.089	.2614	6.791
Yttrium	39	.7255	2.034	.2547	6.792
Strontium	38	.7696	1.974	.2475	6.783
Rubidium	37	.8143	1.919	.2408	6.784
Bromine	35	.9179	1.808	.2272	6.783
Selenium	34	.9790	1.751	.2202	6.775
Arsenic	33	1.0435	1.696	.2135	6.776
Germanium	32	1.1146	1.639	.2067	6.780
Gallium	31	1.1902	1.590	.2002	6.785
Zinc	30	1.2963	1.521	.1922	6.759
Copper	29	1.3785	1.475	.1863	6.774
Nickel	28	1.4890	1.420	.1793	6.766
Cobalt	27	1.6018	1.369	.1730	6.784
Iron	26	1.7396	1.313	.1661	6.780
Manganese	25	1.8892	1.260	.1595	6.787

It is interesting to inquire whether some other quantity connected with x-radiation may not be a linear function of N. If we calculate the velocity of the electron in the x-ray tube required to produce the K emission series from the quantum equation.

$$\tfrac{1}{2}mv^2 = h\nu, \tag{1}$$

assuming that m varies with the velocity according to the law

$$m = \frac{m_0}{\sqrt{1-\beta^2}}, \quad \beta = \frac{v}{c}, \tag{2}$$

where c is the velocity of light, we find that the values of v do not differ

[1] We wish to express our sincere thanks to Professor C. James and to Professor J. L. Howe for sending us some of the rare elements.

from those given by the linear equation

$$v = v_0(N - 3/2), \qquad v_0 = .006783c \tag{3}$$

by as much as 1/5 per cent.

The fifth column in the table contains the values of β, and the sixth, of $\beta_0 = v_0/c$. None of the values of β_0 differ from .006783 by as much as 1/5 per cent., except that for zinc.

We have shown by experiment (*l.c.*) that the critical absorption frequency of rhodium does not differ from the highest frequency in its K emission series by more than about 1/4 per cent. If we calculate the critical velocity v from equations (1) and (2) using for ν the highest frequencies given in Siegbahn's tables of emission lines in the K series, we find that equation (3) represents the velocities with considerable precision for all the chemical elements as far down as and including magnesium ($N = 12$).

The following table contains the data for these chemical elements:

Chemical Element.	Atomic Number.	$\lambda \times 10^3$.	$\sqrt{\nu} \times 10^{-9}$.	Ratio $\beta = v/c$.	$\beta_0 \times 1{,}000$.
Chromium	24	2.067	1.205	.1526	6.782
Titanium	22	2.490	1.097	.1391	6.785
Calcium	20	3.072	.9880	.1253	6.773
Potassium	19	3.446	.9437	.1185	6.771
Chlorine	17	4.391	.8265	.1050	6.774
Sulphur	16	5.014	.7734	.09829	6.779
Phosphorus	15	5.804	.7188	.09137	6.768
Silicon	14	6.755	.6663	.08475	6.778
Aluminum	13	7.982	.6131	.07797	6.778
Magnesium	12	9.471	.5627	.07158	6.817

In conclusion we wish to call attention to the fact that $\frac{1}{2}mv^2$ is not the *relativity* expression for the kinetic energy of the electron. The fact that equation (2) gives the *transverse* mass of the electron suggests that possibly v may represent the velocity of an electron travelling in an orbit. These points will be discussed in another paper.

ON THE X-RAY ABSORPTION FREQUENCIES CHARACTERISTIC OF THE CHEMICAL ELEMENTS.[1]

BY WILLIAM DUANE AND TAKEO SHIMIZU.

SYNOPSIS.

Object.—The critical absorption wave-lengths associated with the K series of x-rays have been measured[2] for all but two of the known chemical elements from manganese ($N = 25$) to cerium ($N = 58$). The object of the experiments described in this paper was to extend the measurements to chemical elements of higher atomic numbers, using the same x-ray spectrometer.

Source of Current.—In the earlier experiments the electric current through the x-ray tube came from a storage battery of 20,000 cells, giving an electromotive force of about 43,000 volts when fully charged. A difference of potential of 43,000 can produce x-rays only a few per cent. shorter than the critical absorption wave-length associated with the K series of cerium, and to make the measurements for chemical elements of larger atomic numbers, having shorter critical absorption wave-lengths, requires a higher voltage applied to the x-ray tube. The authors, therefore, employed a transformer, with a set of condensers and kenotrons attached to it for producing approximately a constant difference of potential. Except for this generating plant the apparatus did not differ from that used in the earlier researches.

Results.—The K-critical absorption wave-lengths were measured for eight of the chemical elements from neodymium ($N = 60$) to lead ($N = 82$) both inclusive. The square roots of the vibration frequencies corresponding to their wave-lengths, when platted against the atomic numbers, indicate a still greater departure from the straight line law than do the values previously obtained for elements of lower atomic numbers.

As has been shown (*l.c.*), the equation

$$v = 0.00678\ (N - \tfrac{5}{2})c \qquad (1)$$

represents to within 1/5 per cent. the velocity of the electrons in the x-ray tube required to produce the K emission series of a chemical element of atomic number lower than $N = 58$, if the velocity is calculated by the quantum relation, using the expression for the transverse mass. The values of v calculated by the same formulas for elements of atomic numbers higher than 58 fall slightly below those given by equation (1). The greatest variations from the equation amount to a little over 1 per cent.

SEVERAL papers have been published[2] describing measurements of the critical absorption wave-lengths associated with the K series of x-rays characteristic of all but two of the known chemical elements from manganese (atomic number $N = 25$) to cerium ($N = 58$) both inclusive.

[1] A paper presented at the Baltimore meeting of the American Physical Society, December 28, 1918.

[2] Blake and Duane, PHYS. REV., Dec., 1917, p. 697; and Duane and Hu, PHYS. REV., June, 1918, p. 488.

In these experiments the electric current flowing through the x-ray tube came from a storage battery of 20,000 cells, giving, when fully charged, a difference of potential of about 43,000 volts. This difference of potential can not produce x-rays much shorter than the K critical absorption wave-length of cerium, and, as the critical wave-length decreases quite rapidly with increasing atomic numbers, it was found impossible to make good measurements for elements of higher atomic numbers than 58.

The authors therefore set up a generating plant similar to that designed by Dr. A. W. Hull. This consisted of a high tension transformer attached to a system of condensers and kenotrons for maintaining the voltage approximately constant. The plant gave us voltages up to about 115,000 volts and we were able to measure the critical absorption wave-lengths associated with the K series of eight of the chemical elements from neodymium ($N = 60$) to lead ($N = 82$).

Except for the generating plant the apparatus did not differ from that used in the earlier researches (*l. c.*), and the same precautions were taken to eliminate errors due to the penetration of the x-rays into the reflecting crystal (calcite), and to the width of the spectrometer's slits.

Curves were drawn for each chemical element similar to those published on page 701 of the PHYSICAL REVIEW for December, 1917, and the angular distances between the mid-points in the sharp drops gave us the grazing angle of incidence, θ, from which to calculate the wave-lengths by the formula

$$\lambda = 6.056 \times \sin\theta \times 10^{-8} \text{ cm.}$$

The following table contains the data. The critical absorption wave-lengths appear in the third column, and the square roots of the corresponding vibration frequencies in the fourth.

The values of the critical absorption wave-lengths given by de Broglie[1] are several per cent. smaller than ours.

Chemical Element.	Atomic Numbers.	$\lambda \times 10^8$.	$\sqrt{\nu} \times 10^{-9}$.	Ratio $\beta = v/c$.	$\beta_0 \times 1{,}000$.
Neodymium	60	.2861	3.237	.3952	6.756
Terbium	65	.2398	3.536	.4282	6.743
Dysprosium	66	.2308	3.604	.4356	6.753
Tungsten	74	.1786	4.097	.4877	6.726
Osmium	76	.1683	4.221	.5003	6.716
Gold	79	.1541	4.411	.5194	6.702
Mercury	80	.1501	4.469	.5252	6.690
Lead	82	.1424	4.588	.5369	6.669

c = the velocity of light.

[1] Journal de Physique, May-June, 1916, p. 161.

On platting the square roots of the vibration frequencies against the atomic numbers it can be seen that they depart from the straight line law still further than do the values previously obtained for chemical elements of lower atomic numbers.

It has been shown (*l.c.*) that the velocity of the electrons in the x-ray tube required to produce the K emission series obeys the law represented by the equation

$$\beta = \frac{v}{c} = \beta_0(N - 3/2) = 0.006783(N - 3/2), \tag{1}$$

for chemical elements up to cerium ($N = 58$), provided that we calculate v from the quantum equation.

$$\tfrac{1}{2}mv^2 = h\nu. \tag{2}$$

In this equation (2) ν is the critical absorption frequency, and m, the transverse mass of the electron, is given by the equation

$$m = \frac{m_0}{\sqrt{(1 - \beta^2)}}. \tag{3}$$

The fifth column in the table contains the values of $\beta = v/c$ calculated by equations (2) and (3), putting $h = 6.555 \times 10^{-27}$, $m_0 = 8.989 \times 10^{-28}$ and $c = 2.9986 \times 10^{10}$.

In the sixth column appear the values of β_0 calculated from the expression $\beta/(N - 3/2)$. The differences between these values of β_0 and that found previously for chemical elements of lower atomic numbers, namely, 0.006783, increase progressively with increasing atomic number, N, the greatest difference being a little more than 1 per cent.

As suggested in a former paper the fact that the *transverse* mass of the electron enters into the equations may mean that v is really the velocity of the electrons in an orbit. If so, the above variation in the value of β_0 may be due to the magnetic effect of the electrons in a ring on each other, and not to a lack of constancy in their angular momenta.

HARVARD UNIVERSITY.

THE RELATION BETWEEN THE INTENSITY OF GENERAL X-RADIATION AND THE ATOMIC NUMBER OF THE ANTICATHODE.[1]

BY WILLIAM DUANE AND TAKEO SHIMIZU.

SYNOPSIS.

Object.—The intensity of general x-radiation is known to be approximately proportional to the atomic weight of the element constituting the anticathode, provided the other conditions are the same. But since the atomic weights are, roughly speaking, proportional to the atomic numbers, it may be that the intensity is proportional to the latter instead of the former, considering the important part which the atomic number plays in the phenomena of characteristic x-radiation. The object of the present experiment was to decide this point.

Experiment.—We used the four successive elements iron, cobalt, nickel and copper because of the advantage that the atomic weights of cobalt and nickel are in the reversed order of their atomic numbers. An x-ray tube of special type, which enabled us to bring any one of the four elements in the focus of a Coolidge cathode without changing the geometrical relations between the focused point and the measuring apparatus, was excited by a large storage battery of available voltage up to 42,000. The intensity of the rays coming out of the tube, which were almost free from characteristic radiations, was measured by means of an ionization chamber and electroscope.

Result.—The intensity was found to be proportional to the atomic number and not to the atomic weight for the above four elements. This was particularly clear because the observed intensities for cobalt and nickel were in the reversed order of their atomic weights.

THE total intensity of general x-radiation regarded as a function of the velocity of the exciting electrons and also as that of the chemical nature of the anticathode was first treated theoretically by J. J. Thomson[2] in 1907. His conclusion, based upon certain assumptions, was that the total intensity should be (I.) proportional to the fourth power of the velocity of the exciting electrons and (II.) independent of the chemical constitution of the anticathode. The first part of the conclusion was a mere anticipation at that time, but it was proved to be true by later experiments of other investigators, especially those of Beatty,[3] who showed that the above law was valid for the general x-radiation. The

[1] A paper presented at a meeting of the American Physical Society, April, 27, 1918.

[2] J. J. Thomson, Electrical Origin of Radiation from Hot Bodies, Phil. Mag., 14, pp. 226–231, Aug., 1907.

[3] R. T. Beatty, Energy of Roentgen Rays, Roy. Soc., Proc., Ser. A. 89, pp. 314–327, Nov. 1, 1913.

second part was, however, contradictory to the known fact that the heavy metals are more efficient radiators of x-rays than the lighter ones. Thomson attributed the cause of this discrepancy between his theory and the fact to the effect of secondary cathode rays.

The first systematic research on the relation between the intensity of general x-radiation and the substance of the anticathode was made by Kaye[1] in 1908. He showed that the intensity was nearly proportional to the atomic weight of the element for about twenty metals he used. This relation was also confirmed later by the experiment of Beatty (*l. c.*).

The work of Moseley on the x-ray spectra of chemical elements showed the great importance of the atomic number in the domain of nuclear phenomena, and the question arises naturally whether the intensity is proportional to the atomic weights of the elements, as shown by Kaye and others, or to their atomic numbers. Since the atomic numbers are nearly proportional to the atomic weights, the answer could not be given without making experiments specially designed for it. The object of the present investigation is to settle this question.

There are three sets of successive elements in the atomic table, the atomic weights and the atomic numbers of which are in the reversed order. For these elements the parallelism between the two quantities is broken, so that they may advantageously be employed in our present inquiry. One of those pairs is that of nickel and cobalt, the atomic weights of which are 58.68 and 58.97 and the atomic numbers 28 and 27 respectively. It is to be noticed that there existed a controversy about the atomic weights of these elements before the important rôle of atomic number was pointed out by Moseley. Thus Barkla and Sadler[2] proposed in 1907 that the atomic weight of nickel might be higher than that of cobalt, despite the fact that the reverse is considered to be established in chemistry. They based their argument upon the penetrating power of secondary x-rays radiated from different elements. At about the same time, J. J. Thomson,[3] observing the intensity of secondary rays from various elements exposed to x-rays, obtained the results which harmonized with the above view. On the other hand, Hacket[4] found that the intensities of secondary rays from those metals

[1] G. W. C. Kaye, Roentgen Rays: Emission and Transmission, Roy. Soc., Phil. Trans., Ser. A, 209, pp. 123–151, Nov. 19, 1908.

[2] C. G. Barkla and C. A. Sadler, Atomic Weight of Nickel, Phil. Mag., 14, pp. 408–422, Sept., 1907.

[3] J. J. Thomson, Secondary Roentgen Radiation, Cambridge Phil. Soc. Proc., 14, pp. 109–114, March 6, 1907.

[4] F. E. Hacket, Atomic Weight of Nickel, Nature, 75, p. 535, April 4, 1907.

exposed to the β-rays of radium were in the same order as their atomic weights, and was opposed to Barkla and Sadler's proposition.

In the present experiment we have measured the intensity of general x-radiation from the four successive elements iron, cobalt, nickel, and copper. In order that the x-rays from all four metals should pass through exactly the same thickness of glass and follow the same path in space, the x-ray bulb indicated in the figure has been used.

Four similar plates of quadrant shape, one of each of the four metals, were soldered to a circular block of copper A, which served as anticathode. The surface consisting of the different metals was carefully turned in a lathe, so that the entire surface made a good plane perpendicular to the axis of a circular copper rod R, which was rigidly attached to A. This rod R could rotate in two copper bearings BB fixed to the glass wall of the tube, but could not slip in the direction of its length. A small piece

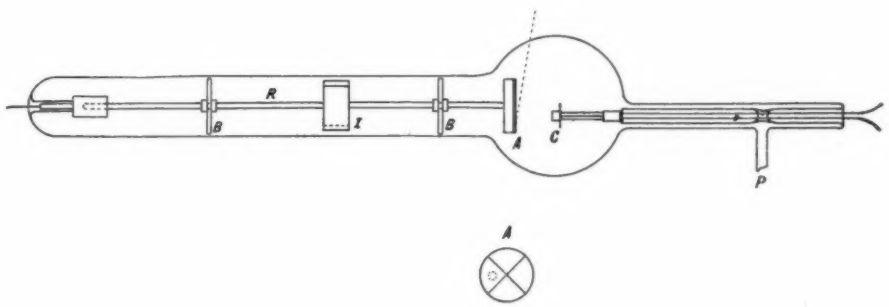


Fig. 1.

of soft iron I, fastened to R made it possible to rotate the anticathode from outside of the tube with an electromagnet.

A Coolidge cathode C was fastened to the tube a little excentric with the axis of R, so that when A was turned around its axis to a suitable position, the cathode rays from it might strike A at the middle of any desired one of the four quadrants, as indicated in the lower figure by a dotted circle.

The side tube P was connected to a Langmuir diffusion pump and a pressure gauge.

As the source of high potential, we employed the 42,000-volt storage battery belonging to the laboratory. The potential across the tube was taken from a suitable number of its sections, and carefully kept constant during observation by adjusting a water rheostat connected in series with the tube. A sensitive static voltmeter made it easy to detect a variation of a few ten thousandths of the applied voltage. The current

through the bulb was also kept constant by adjusting the heating current of the cathode filament.

The x-rays coming out of the bulb sustaining a small angle with the surface, as indicated by a dotted line in the figure were measured by means of an ionization chamber containing saturated vapour of methyl iodide and a Wilson tilted electroscope. Two lead slits placed between the bulb and the ionization chamber defined the solid angle of the measured x-rays. To secure a good constancy of the geometrical relations, the bulb was set up at a distance of about one meter from the nearer slit.

Our object being to compare the radiation from the four metals with one another, observations were taken for them in immediate succession at a constant voltage. But a repetition of one particular observation after an interval of about a month gave almost the same result as was obtained before, indicating that the geometrical relations between the different parts remained unchanged during that time.

A simple calculation showed that the characteristic rays from the metals were almost completely absorbed by the walls of the bulb, so that the observed intensity was due to the general radiation only for all of the metals. The table contains the observed result.

Volts.	Ionization Current (Volts per Sec.).[1]			
	Cu (29).	Ni (28).	Co (27).	Fe (26).
19,140	.0248	.0240	.0227	.0220
21,430	.0386	.0375	.0344	.0336
24,120	.0544	.0532	.0502	.0492
27,300	.0812	.0790	.0792	.0732
30,110	.1088	.1061	.1021	.0981
32,400	.1327	.1295	.1251	.1210
40,870	.238	.231	.223	.214
Volts.	**Relative Intensity, Fe = 1.00 (Calculated from the Above).**			
	Cu.	**Ni.**	**Co.**	**Fe.**
19,140	1.127	1.091	1.032	1.00
21,430	1.149	1.116	1.024	1.00
24,120	1.106	1.081	1.020	1.00
27,300	1.109	1.079	1.041	1.00
30,110	1.109	1.082	1.041	1.00
32,400	1.097	1.070	1.034	1.00
40,870	1.110	1.079	1.042	1.00
Mean.....	1.115	1.085	1.033	1.00
Ratio of atomic nos.	1.115	1.077	1.038	1.00
Ratio of atomic wts.	1.138	1.051	1.056	1.00

[1] The current across the x-ray tube was kept at one milliampere throughout the experiment.

Thus we see that the ratios of the intensities of the general x-radiation produced at the same potential by the different metals are independent of the potential within the limits of errors, at least for the voltages used, and that the intensities are very nearly proportional to the atomic numbers of the elements. This is particularly evident in this case since the order of the atomic weights is not the same as that of the atomic numbers. The intensity follows the order of the atomic numbers and not that of atomic weights.

HARVARD UNIVERSITY.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE PHILADELPHIA MEETING, OCTOBER 10 AND 11, 1919.

THE ninety-ninth meeting of the American Physical Society was held in Philadelphia on October 10 and 11, 1919. On Friday, October 10, there were three sessions, arranged by the Committee on Technical Physics. These were joint meetings with the American Institute of Electrical Engineers and were held at the Hotel Bellevue-Stratford.

In the session of Friday morning, at which President Ames of the American Physical Society presided, the following program was presented:

A Discussion on the Present Status of Theories of Atomic Structure:

I. Introductory Remarks on Present Theories. SAUL DUSHMAN.

II. Special Applications of the Octet Theory. IRVING LANGMUIR.

III. Spectroscopic Evidence Regarding Atomic Structure. F. A. SAUNDERS.

IV. Bearings of High Pressure Phenomena on the Problem of the Constitution of Matter. P. W. BRIDGEMAN.

On Friday afternoon, under the auspices of the American Institute of Electrical Engineers, the following program was presented:

The Arrangement of Atoms in Metals. A. W. HULL. An X-ray study of crystal structures, a demonstration with models of crystals.

The Oscillating Vacuum Tube as a Generator of Electrical Power. J. H. MORECROFT AND H. T. FRIIS.

Electromagnetic Induction. S. J. BARNETT.

Piezo Electric Effect. A. M. NICHOLSON. (Demonstration.)

On Friday evening there was an informal dinner in the Clover Room of the hotel, followed by a joint meeting of the Institute and the Physical Society, at which two papers were presented as follows:

The Indispensability to each other of Pure and Applied Science. H. A. BUMSTEAD.

Pure Science and Industrial Research. J. J. CARTY.

On Saturday, October 11, 1919, there were two general sessions of the Physical Society held, by invitation, in the auditorium of the Leeds and Northrup Company, Vice-President Lyman presiding. In the intermission the members availed themselves of the invitation to inspect the laboratories of the Leeds and Northrup Company. A complimentary luncheon was provided by our hosts.

The program began with two papers presented by invitation of the Committee, as follows:

Research Development and Technical Control in a Small Industry. M. E. LEEDS.

Some Research and Development Problems of the Leeds & Northrup Co. R. B. SMITH.

The regular program consisted of eleven papers contributed by members in general, two of these being read by title, as follows:

A Precision Potentiometer. ARTHUR W. GRAY.

On Establishing of the Absolute Temperature Scale. FREDERICK G. KEYES.

The Crystal Structure of Ferro-Magnetic Metals. ALBERT W. HULL.

Hydrogen Overvoltage. D. A. MACINNES, LEON ADLER AND A. W. CONTIERI.

Note on a Method of Measuring the Decrease of Electron in Thin Films by Means of Characteristic X-Rays. (Read by title.) BARGEN DAVIS.

The Application of Motion Picture Photography to Ballistics and Airplane Performances. F. C. BROWN.

Unipolar Induction. W. F. G. SWANN.

Magnetic Resistance Effects in Films of Bismuth. F. K. RICHTMYER AND L. F. CURTISS.

The Spectral Photoelectric Sensitivity of Molybdenite as a Function of the Applied Voltage. (Read by title.) W. W. COBLENTZ and H. Kahler.

Ionization and Resonance Potentials for Electrons in Vapors of Lead and Cadmium. F. L. MOHLER, PAUL D. FOOTE AND H. F. STIMSON.

The Calculation of Audion Constants. R. W. KING.

At the meeting of the Council held on October 11, 1919, the following elections were made: *elected to regular membership*, L. H. Adams, E. Ferrari, M. LaRosa, F. R. Moulton, S. Nakamura, O. Veblen; *elected to associate membership*, A. K. Aster, F. I. Bernhard, E. C. Brady, I. C. Cornog, L. F. Curtiss, R. C. Duncan, A. L. Ellis, J. L. Fearing, Miss M. K. Frehafer, J. C. Karcher, R. H. Kent, V. F. Lenzen, F. H. McBerty, W. C. Pomeroy, Miss J. A. Rodman, T. G. Seidell, A. F. Wagner, W. R. Williams; *transferred from associate to regular membership*, J. H. Morecroft, T. Takamine, E. D. Tillyer.

DAYTON C. MILLER, *Secretary*.

THE APPLICATION OF MOTION-PICTURE PHOTOGRAPHY TO BALLISTICS AND AIRPLANE PERFORMANCES.[1]

BY F. C. BROWN.

THIS paper shows, with the aid of motion pictures and lantern slides, the results obtained by photographing the reflected image of the Handley-Page airplane simultaneously with the Mark IV bomb. These

[1] Abstract of a paper presented at the Philadelphia meeting of the American Physical Society, October 11, 1919.

photographs were taken from the airplane with the motion-picture camera placed directly above the bomb.

From an enlarged reproduction of the motion picture film, of magnification of six, was calculated the axis of the bomb relative to the trajectory tangent from the time it left the airplane until it struck the water 2,500 feet below. Also, the trail angle of the bomb back of the reflected image of the plane was calculated to an accuracy of about .03 degree. The results show very clearly the free period of the bomb throughout the trajectory, and also a close correlation between the lag and lead of the bomb and the position of the bomb on the trajectory. The so-called skidding effect is measured quite accurately.

The altitude of the airplane is calculated from the size of the reflected image on the film to a probable accuracy of about 15 feet, while the distance that the bomb has fallen is calculated from the diameter of the bomb image on the film to a probable accuracy of 100 feet.

The rolling and pitching of the airplane carrying the bomb at the time of release and when the explosive wave hits the plane is shown to an accuracy of a few hundredths of a degree. The free period of the plane during flight is less than .5 degree on either side of the mean flying position. When the explosive wave hits the airplane, there is a transverse displacement of the wings of nearly 1 degree, but the explosive wave, when the bomb explodes above water, does not affect the pitching of the airplane by as much as .1 degree.

ORDNANCE DEPARTMENT,
BUREAU OF STANDARDS.

THE CALCULATION OF AUDION CONSTANTS.[1]

BY R. W. KING.

GIVEN the characteristic equation of an audion as

$$I_p = \kappa\left(\frac{E_p}{\mu} + E_c\right)^{\eta},$$

it has been found possible to deduce expressions for the constants μ and κ from theoretical considerations which agree closely with experimental results. The calculation has been carried out for four classes of audions.

1. The structure with plane-parallel elements in which the filament is symmetrically placed between grids and plates;

2. The structure with cylindrical anode and grid and a coaxial strand of filament;

3. The structure with cylindrical anode and grid and several parallel strands of filament arranged on an internal coaxial cylinder;

4. The structure similar to Class 3 except that the strands of filament lie on an external coaxial cylinder. These calculations result in the following formulas.

[1] Abstract of a paper presented at the Philadelphia meeting of the American Physical Society, October 11, 1919.

Class 1.—Given α the distance from plate to grid, β the distance from grid to filament, n the number of grid wires per unit length, r the radius of the grid wires, and A the total plate area,

$$\mu = \frac{2\pi r n}{\log_e \frac{1}{2\pi\alpha n}},$$

$$\kappa = 2.33 \times 10^{-6} \frac{A}{\sqrt{\alpha+\beta}} \left(\frac{\mu}{\alpha + (\mu+1)} \right)^{3/2}.$$

In this and the following values for κ it has been assumed that the exponent η has the value 3/2. This assumption in general gives good numerical agreement.

Class 2.—Given R_p, R_g, R_f the radii of plate, grid and filament respectively, and l the length of the structure,

$$\mu = \frac{\left(2\pi R_g^2 \frac{1}{R_g} - \frac{1}{R_p} \right) n}{\log_e \frac{1}{2\pi r n}},$$

$$\kappa = 14.65 \times 10^{-6} \frac{l R_p^{1/2} \mu^{3/2}}{[(R_p - R_g) + R_g(\mu+1)]^{3/2}}.$$

Classes 3 and 4.—The expression for μ is that given under Class 2. The expression for κ is

$$\kappa = 14.65 \times 10^{-6} \frac{R_p l \mu^{3/2}}{\sqrt{\pm(R_p - R_f) \pm (R_p - R_g) \pm (R_g - R_f)(\mu+1)^{3/2}}}$$

where the signs are taken in such a way as to make all terms positive.

It is evident from the above expressions for μ and κ that these two quantities remain unchanged so long as the *relative* dimensions of any structure remain the same.

The accompanying table shows the numerical agreement obtained with some tubes of Class 1. All dimensions are in centimeters and currents in milliamperes.

TABLE.

α.	β.	n.	r.	A.	μ'.		E_p.	E_g.	I_p.	
					Calc'd.	Obs'd.			Calc'd.	Obs'd.
0.228	0.169	2.52	0.01	12.1	1.97	1.90	120	−10.7	70	64
.466	.169	3.45	.01	12.1	6.6	6.1	300	0	79	69
3.17	.317	6.29	.01	22.6	13.6	13.0	700	0	112	99

RESEARCH LABORATORIES OF THE AMERICAN TELEPHONE AND TELEGRAPH COMPANY AND THE WESTERN ELECTRIC COMPANY, INC.,
NEW YORK CITY.

IONIZATION AND RESONANCE POTENTIALS FOR ELECTRONS IN VAPORS OF LEAD AND CALCIUM.[1]

BY F. L. MOHLER, PAUL D. FOOTE AND H. F. STIMSON.

MEASUREMENTS of electron currents in three electrode vacuum tubes of the type previously described have been made in vapors of lead and calcium.

The lead and calcium were boiled in porcelain tubes at temperatures of about 1000° and 900° respectively. Current voltage curves in lead showed a resonance potential of 1.26 volts and an ionization potential of 7.93 volts. Applying the quantum relation $Ve = h\nu$ we find that 1.26 volts corresponds within experimental error to the frequency of a strong infra-red spectrum line at $\lambda = 10{,}291$ Å. giving a theoretical value of the resonance potential 1.198 volts.

In calcium two resonance potentials were found at 1.90 volts and at 2.85 volts of which the first is the most prominent. Ionization occurred at 6.01 volts. The ionization potential corresponds to the limit of the principal series $1.5S\lambda = 2027$ Å. giving as the theoretical value $V = 6.081$ volts. The first resonance is determined by the line $1.5\ S - 2p_2$. $\lambda = 6572.78$ Å. $V = 1.877$ volts. The second resonance corresponds to the line $1.5S - 2P$. $\lambda = 4226.73$ Å. $V = 2.918$ volts.

The spectral relations of the first resonance potential and ionization potential are analogous to the relation found with other metals in this group. Work of other observers shows that both the lines $1.5S - 2P$ and $1.5S - 2p_2$ appear below the ionization potential in most metals of this group.

BUREAU OF STANDARDS
October 1, 1919.

THE SPECTRAL PHOTOELECTRIC SENSITIVITY ON MOLYBDENITE AS A FUNCTION OF THE APPLIED VOLTAGE.[1]

BY W. W. COBLENTZ AND H. KAHLER.

IN a previous investigation it was observed that the spectral photoelectric sensitivity of molybdenite is confined practically within three spectral bands, the maxima of which are separated by equal intervals, when plotted in terms of frequency instead of wave-lengths.

The present investigation was undertaken in order to test the validity of this frequency relation, using for the purpose a quartz prism which gave twice the dispersion formerly obtained with a fluosite prism. The sample of molybdenite was soldered to copper wire terminals, and operated in an evacuated chamber as in previous work. The photoelectric substance, the dry battery and the d'Arsonval gavanometer were joined in series. The deflection caused by the

[1] Abstract of a paper presented at the Philadelphia meeting of the American Physical Society, October 11, 1919.

small dark current was overcome by rotating the suspension head of the galvanometer.

The results obtained with this apparatus appear to verify the previous observations, indicating that the frequency maxima are separated by equal intervals, which decrease with temperature; the arbitrary wave-number being $n = 40$ at 25° C. and $n = 30$ at $-$ 175° C.

A New Voltage Phenomenon.—A photo-negative action (resistance increase) has been observed in certain samples of selenium,[1] when exposed to the total radiation from an incandescent lamp. A similar spectral photo-negative response was observed in some samples of stibnite,[2] when exposed to radiations of wave-lengths less than $\lambda = .657\ \mu$. But heretofore no one appears to have observed that, for wave-lengths less than about 0.65 μ, the nature of the photoelectric response depends upon the vo'tage applied to the substance under test.

We have found in some samples of molybdenite that, for the visible spectrum extending to about $\lambda = 0.647\ \mu$, the electric response is photo-positive or photo-negative, depending upon the voltage applied to the terminals of the receiver. For wave-lengths greater than $\lambda = 0.647\ \mu$, the photoelectric response was observed to be photo-positive whatever the applied voltage.

The region of transition in the spectrum, in which the action changes from photo-negative to photo-positive, is very narrow,—less than 9 Å. U.

The critical voltage is very small as may be inferred from the fact that an increase of 1.3 volts (additional dry battery) changed a positive-negative galvanometer deflective of $\pm$ 1 cm. into a purely negative deflection of $-$ 24 to $-$ 26 cm., which is the photo-negative response under discussion.

There seem to be two contending forces acting. The one which causes the photo-positive response acts quickly and prevails on low voltage. The photo-negative action builds up more slowly and is predominant on high voltages. As a result of these two forces, for certain applied voltages, on exposing the the molybdenite receiver to light of wave-lengths less than 0.647 μ, the galvanometer deflection is first positive, then decreases in value (and may even become negative) when, on shutting off the light stimulus, there is a further deflection in the negative direction, after which the deflection returns to the original zero scale reading.

For example, using the wave-length $\lambda = .5876\ \mu$ as a light stimulus, and applying a potential of 10 volts, the galvanometer deflection was almost entirely positive. On 20 volts the deflection was partly positive and partly negative. On 29 volts the positive response was almost eliminated, and the negative response prevailed. On 33 volts the galvanometer deflection was entirely negative and eight times as large as the observed maximum purely positive deflection, for 10 volts. In another test, at a slightly lower temperature ($-$104° C.) which raises the critical voltage, changing from 35.6 volts to 37 volts transformed the positive-negative deflection of $\pm$ 5 mm. into a negative deflection of $-$ 230 mm.

[1] Ries, Phys. Zeit., 9, p. 569; 1908. Brown, PHYS. REV., 33, p. 1; 1911.

[2] Elliot, Phys. Rev., (2) 5, p. 62; 1915.

These tests were carried out at $-100°$ C. to $-178°$ C. It is of interest to note that the photo-positive action is the same as a resistance decrease caused by a rise in temperature of the material, while the photo-negative action is similar to the building up of a counter-electromotive force (the electrolytic action) previously observed in silver sulphide. An equal energy spectrum was used for the radiation stimulus and if this phenomenon were the result of heating and of electrolytic action, then the photo-positive response should occur in the short wave-lengths where the absorption is greatest, and the photo-negative response should occur in the long wave-lengths where the photoelectric activity is the greatest. This is just the reverse of what has been observed.

No explanation of this phenomenon is attempted at this time. Even if it is "only a gas effect," "electrolytic action" or "surface charge," it is unique in being selective as to the wave-length of the exciting radiation and in being photopositive or photo-negative, depending upon the applied voltage. It therefore requires further investigation.

WASHINGTON, D. C.,
October 2, 1919.

MAGNETO-RESISTANCE EFFECTS IN FILMS OF BISMUTH.[1]

BY F. K. RICHTMYER AND L. F. CURTISS.

THERE is a disagreement among previous investigators concerning the magneto-resistanee effects in films of bismuth obtained by cathodic sputtering. The present work has been undertaken as a preliminary attempt to obtain further information on this subject, which has an important bearing on the electron theory of metallic conduction. Films of various thickness were sputtered from chemically pure bismuth. It has been found that if especial care is taken to avoid heating these films during the sputtering, by allowing the process to go on only intermittently at short intervals, there is little or no change produced in the resistance of such films when placed in a transverse magnetic field of 16,000 gauss. Heating the films will restore this property. If the films are heated to a temperature near the melting point of the bismuth and allowed to cool and this process repeated several times, after each heating the film exhibits a greater change of resistance after each heating than before it, until a certain upper limit is reached, beyond which further heating, if the previous maximum temperature is not exceeded, has little effect. Likewise, heating to successively higher temperatures, beginning at a fairly low maximum and gradually approaching the melting point of the metal in each successive heating, restores the property under investigation to a greater and greater degree. Heating to temperatures under 150° has very little effect towards restoring this property, while heating to temperatures above this value has a correspondingly larger effect. If the film is strongly heated while being sputtered the initial increase of resistance in the magnetic

[1] Abstract of a paper presented at the Philadelphia meeting of the American Physical Society, October 11, 1919.

field is comparatively large and further heating produces only slight increases in this initial value. The heating produces no observable change in the negative temperature coefficient of resistance of these films, which was carefully measured before and after each heating.

UNIPOLAR INDUCTION.[1]

BY W. F. G. SWANN.

THE paper forms an enquiry as to the extent to which the so called "Moving Line Theory" of electromagnetic induction is the equivalent of the Maxwell-Lorentz Theory. The latter theory gives the force E on a fixed unit of charge in the form:

$$E = -\frac{l\partial U}{c\partial t} - \frac{\partial \psi}{\partial x}, \qquad (1)$$

where U is the Maxwellian vector-potential, and ψ is the electro-static potential.

Consider the case of an infinitesimal amperian whirl of negative electricity on which is superposed a distribution of positive electricity which just cancels the electrostatic effect of the negative when the whirl is without translatory motion. As shown in the paper, the theory of relativity requires that, when the whirl is set into translatory motion perpendicular to its axis, there shall arise an electrostatic potential which is the equivalent of that of an electric doublet placed at the center of the whirl. The total electric field is thus composed of the field due to this doublet together with a part depending upon the time rate of change of the vector-potential; and, the inclusion of this electrostatic potential, which comes into existence by a rearrangement of the electric density in the whirl, makes the Maxwell-Lorentz theory agree with the "moving line theory" for this case.

Now in the case of a rotating magnet, the various elementary whirls partake of a translatory motion, an acceleration towards the center of rotation of the the magnet, and the axes of the whirls may also undergo a rotary motion. To the extent that we may assume the constitution of the whirls to be unaltered by the motion except to the extent indicated above, we may calculate the field which would be produced by a symmetrical insulating magnet of unit specific inductive capacity rotating about its axis of magnetization, by applying the "moving-line theory" to each of the elements which constitute the magnet. In doing this, however, only the translatory motion of the elements which constitute the magnet. In doing this, however, only the translatory motion of the elements must be considered. We must exclude any idea of movement of the magnetic lines of force as a result of rotary motion of the elements; for, neither on the basis of physical intuition, nor on the more exact basis of the Lorentzian equations are we justified in expecting any field as a result of mere rotation of the frame of the whirl about its own axis.

[1] Abstract of a paper presented at the Philadelphia meeting of the American Physical Society, October 11, 1919.

The fundamental difference between the "moving-line theory" as it is ordinarily applied, and the theory in the form in which it would be applied lies in the fact that, in the former, the motion attributed to the magnetic field is the same as if the magnetic lines from the elementary whirls were pinned down at places where they pass through the axis of rotation of the magnet. In other words, the motion of the lines as is calculated as though they partook of the rotational velocity of the *frame* of the whirl about its own axis as well as of the translatory velocity of the whirl. As we have remarked, the former part should be omitted; and, when this is done, we have the "moving-line theory" in its proper form.

In a symmetrical rotating system, the vector-potential does not change with the time so that the second member of the right-hand side of (1) is the only term which contributes to E. The effect of this term is equivalent to a set of electric doublets with their axes radial. We must suppose one doublet for each molecular magnet, and the moments of the doublets are proportional to their distances from the axis of rotation in the case of a uniformly magnetized system.

By Poissen's mathematical theorem on polarized media, the field due to the doublets is equivalent to that of a certain fictitious distribution of electricity throughout the volume, and a distribution of electricity of opposite sign and of equal total amount over the surface.

In a conducting magnet we have two additional facts to consider. The force on a unit of electricity which is in motion with velocity V is given by:

$$F = E + \frac{[TH]}{C} \qquad (2)$$

If for the moment we ignore the second member of this force, it will be necessary for the potential to be constant and the field zero throughout the magnet. This can only result by the appearance of real surface and volume distributions which, at each element, are equal and opposite in amount to the fictitious charge corresponding to the element. The result is then a complete cancellation of the fields due to these fictitious charges, not only within the magnet but at all points outside.

If we now introduce the second member of the force F into the consideration, we see that, owing to the motion of the substance of the magnet in its own magnetic field, an electromotive intensity will arise, and this will bring into existence an electrostatic distribution whose field, at all points within the magnet, just cancels the electromotive intensity which caused it. The electrostatic distribution arising in this way is then the only factor which ultimately contributes to the field outside the magnet. Its effect is of course always completely shielded off from any space from which the magnet is shielded by an earthed shield which does not participate in the rotation.

The paper concludes with an application of the above discussion to certain typical experiments, and an extension of the ideas to the case of an elementary whirl rotating as a whole about an axis in itself, other than the primary axis of the whirl.

DEPARTMENT OF PHYSICS, UNIVERSITY OF MINNESOTA.

NOTE ON A METHOD OF MEASURING THE DECREASE OF VELOCITIES IN THIN FILMS BY MEANS OF CHARACTERISTIC X-RAYS.[1]

BY BERGEN DAVIS.

IT is desired to point out briefly that the phenomenon of characteristic X-rays furnishes a convenient and probably accurate means of measuring the decrease of electron velocities in penetrating metal surfaces.

A method of applying characteristic X-rays for this purpose was described in the PHYSICAL REVIEW for June, 1918. The method here described is much simpler than the one suggested.

The method under consideration depends on the fact that there is a definite velocity of electron impact necessary to excite characteristic X-rays of any element.

If a thin film of the metal to be investigated be deposited on the surface of another metal giving characteristic X-radiation within the range of observation, the characteristic radiation will not be excited in the under metal unless the electrons have penetrated the film of known thickness above and reach the metal with the required velocity. This limiting velocity is already known. The velocity with which the electrons enter the film can be calculated from the voltage applied.

The velocity is thus determined on each side of a metallic film of known thickness.

A difficulty that may be encountered is a possible alteration in the thickness of the metal film both by the mechanical action and also by the slow evaporation of the metal as the target becomes heated.

The decrease of electron velocities in penetrating gold and aluminium foils has been measured by the direct magnetic deflection method by Whiddington. The decrease seemed to follow the law:

$$U_x^4 = V_0^4 - ax.$$

The constant a did not appear to depend in any simple way on the properties of the metals. It is very desirable that this constant should be determined for many metals as it enters largely into all theories of X-ray emission.

DEPARTMENT OF PHYSICS,
COLUMBIA UNIVERSITY.

HYDROGEN OVERVOLTAGE.[1]

BY D. A. MACINNES, LEON ADLER AND A. W. CONTIERI.

HYDROGEN overvoltage is due, primarily, to a layer of supersaturated dissolved hydrogen in the electrolyte surrounding an electrode. If, however, the electrode can hold nuclei of gaseous hydrogen on its surface the supersaturation will not rise to high values, since a portion of the hydrogen

[1] Abstract of a paper presented at the Philadelphia meeting of the American Physical Society, October 11, 1919.

will then go into the gaseous phase. Metals with small adsorbtive powers hold small nuclei and have high overvoltages. This explanation of overvoltage is supported by experimental evidence as to the presence of gaseous nuclei, and by observations of fluctuations of overvoltage during the evolution of a single minute bubble. Further, the relation between the radius of the evolved bubbles γ, the surface tension γ, the pressure p and the overvoltage E has been shown experimentally to be, for platinum electrodes,

$$2FE = \frac{3RT}{pr\gamma}.$$

The work of Goodwin and Wilson 'unpublished' has shown that the change of overvoltage with pressure follows, nearly quantitatively, the relation given above, for electrodes of nickel, lead and mercury, *i.e.*, the overvoltage rises with a decrease of pressure.

A study has also been made of the effect of the change of gaseous pressure on several chemical and electrochemical processes involving the evolution of hydrogen. The changes in rates of reaction and in reaction efficiencies were found in each case to be in the directions which follow from the change of hydrogen overvoltage with pressure, *i.e.*, a decrease of pressure produces (*a*) a decrease of the rate of solution of metals in electrolytes, (*b*) an increase in the efficiency of reductions by metals, and (*c*) an increased efficiency of metal deposition.

THE CRYSTAL STRUCTURE OF FERRO-MAGNETIC METALS.[1]

BY ALBERT W. HULL.

IT is well known that ferro-magnetism is not a specific property of the atom, that is, the atoms of ferro-magnetic materials are, in general, ferro-magnetic only when in pure metallic condition and between definite limits of temperature. The relative spacing of the atoms is apparently assential. It might have been anticipated, therefore, that ferro-magnetic metals would have the same crystal structure. That this is not the case is evident from the following table, which shows the type of lattice and distance between atoms of these metals as determined from powder photographs. (For method see PHYSICAL

TABLE.

Substance.	Type of Lattice.	Lattice Constant [Side of Elementary Cube or Triangle.]	Distance Between Nearest Atoms.
Chromium...	Centered Cubic	2.91	2.52
Iron.....	Centered Cubic	2.86	2.47
Cobalt	Hexagonal "Close-packed"	2.52	2.52
	Face-Centered Cube	3.57	2.52
Nickel.....	Face-Centered Cube	3.54	2.50

[1] Abstract of a paper presented at the Philadelphia meeting of the American Physical Society, October 11, 1919.

REVIEW, 10, 661–696, 1917. The full data will be published elsewhere in this journal.)

It will be observed that the ferro-magnetic metal, nickel, has the same crystal structure as copper, while cobalt appears in two forms—one like copper and one like magnesium. Neither is like iron. Chromium, on the other hand, which is not ferro-magnetic has a centered cubic arrangement like iron. Manganese has not yet been accessible in sufficiently pure form to allow of its determination.

It is evident, therefore, that ferro-magnetism does not depend upon any particular arrangement of atoms. It is much more likely that it depends upon the distances between the atoms, which would explain the fact that this property is lost when the temperature is increased beyond a definite value.

RESEARCH LABORATORY,
GENERAL ELECTRIC CO.

A PRECISION POTENTIOMETER.[1]

BY ARTHUR W. GRAY.

THE potentiometer forming the subject of this communication has been designed especially for the accurate measurement of small electromotive forces, such as those produced by thermocouples; but its range can be extended by employing a volt-box in the usual way.

The instrument possesses the following advantages:

1. The galvanometer circuit is practically free from parasitic thermal electromotive forces.

2. An eliminating switch provides for easily correcting the galvanometer deflections for any parasitic electromotive forces that may happen to exist.

3. There are no sliding contacts in the galvanometer circuit.

4. The resistances of the sliding contacts in the battery circuits are negligible.

5. The operation of balancing the electromotive force to be measured does not change the resistances in series with either the galvanometer or the batteries; consequently, it does not change the galvanometer sensitivity.

6. A three-point switch provides a means of reducing the galvanometer sensitivity to 0.1 or 0.01 of the greatest sensitivity used.

7. The galvanometer is critically damped for each sensitivity when balancing either the standard cell or the electromotive force to be measured, and also when the eliminating switch is closed.

8. The range of the instrument can be multiplied by 10 by shifting switches operable by a single handle.

9. An electromotive force can be balanced as closely as with a slide-wire potentiometer without introducing any of the disadvantages of a slide wire.

10. The potentiometer can be used either as a null-method instrument or as a deflection instrument.

[1] Abstract of a paper presented at the Philadelphia meeting of the American Physical Society, October 11, 1919.

11. While the manipulation is as convenient as that of a potentiometer in which all the essential parts are contained in one box, the instrument is made up of parts which can readily be used separately for other purposes.

12. Its cost is low in comparison with that of other potentiometers yielding the same accuracy.

For some time past there has been in regular use a potentiometer of this type which measures any E.M.F. up to 10,000 microvolts with a precision of one microvolt. A galvanometer deflection of 5 millimeters indicates 1, 10, or 100 microvolts, depending upon the position of a switch. The operating current is furnished by dry cells.

PHYSICAL RESEARCH LABORATORY,
THE L. D. CAULK COMPANY,
MILFORD, DEL.,
Sept. 19, 1919.

ON ESTABLISHING OF THE ABSOLUTE TEMPERATURE SCALE.[1]

BY FREDERICK G. KEYES.

BY employing the equation of state whose physical basis was set forth by the writer in the Proceedings of the National Academy for 1916, corrections have been made for the constant pressure nitrogen thermometer. The hydrogen and helium equations have in addition been employed to compute the position of the ice point on the absolute scale. The scale corrections are practically the same as those arrived at by Buckingham by the use of the available Joule-Thomson coefficients. The ice point is found, employing the recent Dayand Clement Pt-Ir bulb constant volume nitrogen expansion coefficients, to be about 273.14.

The form of equation for one type molecule systems is a linear function of the temperature at constant volume and accordingly the constant volume thermometer readings would lie on the absolute scale. The recent work of Eumorfopoulos on the boiling point of sulphur employing a quartz bulb with the Callendar constant pressure thermometer is free from the disturbing effect inherent to the porcelain and glass bulb previously employed. The values obtained by Eumorfopoulos, corrected to the absolute scale, lead to a mean value for the S.B.P. of 444.54 whereas the mean obtained with the constant volume nitrogen thermometer is 444.52.

[1] Abstract of a paper presented at the Philadelphia meeting of the American Physical Society, October 11, 1919. Contribution from the Research Laboratory of Physical Chemistry of the Massachusetts Institute of Technology.

Index to Volume XIV, Series II.

A.

Absolute Temperature Scale, On Establishing of the, *Frederick G. Keyes*, 542.

Absorption, Fluorescence and, of the Uranyl Sulphates, *E. L. Nichols and H. L. Howes*, 293.

Absorption, The, of High Frequency Radiation. The Size and Shape of the Electron, *Arthur H. Compton*, 247.

Absorption, Critical, On the, and Characteristic Emission X-Ray Frequencies, *William Duane* and *Kang-Fuh Hu*, 369.

Absorption, Evolution and, of Gases by Glass, *D. Ulrey*, 160.

Absorption, Fluorescence and, of the Uranyl Acetates, *E. L. Nichols, H. L. Howes* and *Frances G. Wick*, 201.

Absorption Effects with the Electric Furnace as Related to Temperature Classification, *Arthur S. King*, 270.

Absorption of Gases, Studies of the, by Charcoal. I. Variations due to Heat Treatment, Preëquilibrium Effects, *Harvey B. Lemon*, 281.

Acetylene Flame, Note on the Distribution of Energy by the Visible Spectrum of a Cylindrical, *W. W. Coblentz*, 168.

Acetylene Flame, On the Distribution of Energy in the Visible Spectrum of an, *E. P. Hyde, W. E. Forsythe* and *F. E. Cady*, 379.

Adams, Leason H., A Furnace Temperature Regulator, 44.

Temperature Distribution in Solids During Heating or Cooling, 99.

Adler, Leon, Hydrogen Overvoltage, 539.

Adsorption of Gases, Studies of the, by Charcoal, II, *Harvey B. Lemon* and *Kathryn Blodgett*, 394.

Airplane Inclinometers, *David L. Webster*, 161.

Aircraft Engines, A Study of the Velocity of Flame Propagation in the Cylinders of, *D. MacKenzie* and *R. K. Honaman*, 169.

Airplane Fabrics, An Investigation of the Physical Properties of Cotton, *R. W. Duncan*, 194.

Aircraft, Location of, by Sound, *G. W. Stewart*, 166.

Aluminium Target, On the Spectrum of X-Rays from an, *William Duane* and *Takeo Shimizu*, 389.

American Physical Society; Abstracts, 159, 261, 450, 531. Minutes, 152, 260, 530.

Anderson, J. A., The Stark Effect for Metals in the Ultra-Violet, 270.

Antenna, Electric Wave Transmission Formulas for, and Coil Aërials, *J. H. Dellinger*, 180.

Arc, A Study of the Relative Intensities of Spectrum Lines in Different Regions of the, as Compared with their Behavior in other Sources, *Paul W. Merrill*, 271.

Argon, Resonance and Ionization Potentials for Electrons in the Monatomic Gases, Neon and Helium, *H. C. Rentschler*, 503.

Artillery Projectiles, Progress in Design of, *O. Veblen*, 167.

Aspirating Pump, The Bunsen, and the Bernouilli Principle, *Will C. Baker*, 228.

Audion Constants, The Calculation of, *R. W. King*, 532.

Axis of Symmetry, The Stability of a Rotating Parachute or Falling Body Rotating About a Vertical, *H. Bateman*, 269.

B.

Baker, Will C., Displacement from the Apparent Vertical in Free Fall, 352.

Baker, Will C., The Bunsen Aspirating Pump and the Bernouilli Principle, 228.

Ballistic Problems, The Use of the Oscillograph for the Study of, *H. L. Curtis* and *R. C. Duncan*, 183.

Bernouilli Principle, The Bunsen Aspirating Pump and the, *Will C. Baker*, 228.

Baroscope, A, for Measuring the Pressure in Explosion Waves, *Dayton C. Miller*, 450.

Bates, S. J., The Vapor Pressures of Hydrogen Chloride and Hydrogen Bromide above their Aqueous Solution, 271.

Bateman, H., The Stability of a Rotating Parachute or Falling Body Rotating about a Vertical Axis of Symmetry, 269.

Benton, A. F., The End Correction in the Determination of Gas Viscosity by the Capillary Tube Method, 403.

Bismuth, Magneto-Resistance Effects in Films of, *F. K. Richtmer* and *L. F. Curtiss*, 536.

Birge, Raymond T., The Most Probable Value of the Planck Constant *h*, 361.
Blodgett, Kathryn, Studies of the Adsorption of Gases by Charcoal, II, 394.
Breit, G., Inductance, Capacity and Resistance of Coils at Radio Frequencies, 170.
Bridgman, P. W., A Critical Thermodynamic Discussion of the Volta, Thermoelectric and Thermionic Effects, 306.
Brown, F. C., The Application of Motion-Picture Photography to Ballistics and Airplane Performances, 531.
Bunsen Aspirating Pump, The, and the Bernouilli Principle, *Will C. Baker*, 228.

C.

Cady, F. E., On the Distribution of Energy in the Visible Spectrum of an Acetylene Flame, 379.
Camouflage Detection, Photographic Method of Detecting Changes in a Complicated Group of Objects with Especial Reference to, *M. H. Stillman*, 173.
Camouflage, The Spectral Transmission of Filters used to Detect, or Improve Visibility, *K. S. Gibson, E. P. T. Tyndall* and *H. J. McNicholas*, 261.
Capacity, A Note on the Comparison of Inductances, or of an Inductance and a, by an Electrometer Method, *Alva W. Smith*, 356.
Capacity, Inductance, and Resistance of Coils at Radio Frequencies, *L. E. Whittemore* and *G. Breit*, 170.
Capillarity, The Movement of Moisture in Soil by, *Willard Gardner*, 276.
Capillarity Tube Method, The End Correction in the Determination of Gas Viscosity by the, *A. F. Benton*, 403.
Carnot's Principle, The Measurement of Heat and the Scope of, *Arthur C. Lunn*, 1.
Cathode-Ray Tube, Applications of the, in Radio Work, *L. E. Whittemore* and *L. M. Hull*, 266.
Characteristic Emission, On the Critical Absorption and, X-Ray Frequencies, *William Duane* and *Kang-Fuh Hu*, 369.
Charcoal, Studies of the Absorption of Gases by, I. Variations due to Heat Treatment, Preëquilibrium Effects, *Harvey B. Lemon*, 281.
Charcoal, Studies of the Adsorption of Gases by, II, *Harvey B. Lemon* and *Kathryn Blodgett*, 394.
Chronograph, A New, for Measurement of Projectile Velocities and Short Time Intervals, 159.
Circuit Problems, The Solution of, Mathematical Methods of Investigation Resulting from the Application of Fourier's Integral, 115.
Coblentz, W. W., Comments on Spectral Radiation Formulæ, 267.
Coblentz, W. W., Note on the Coefficient of Total Radiation of a Uniformly Heated Enclosure, 174.
Note on the Distribution of Energy by the Visible Spectrum of a Cylindrical Acetylene Flame, 168.
The Spectral Photoelectric Sensitivity on Molybdenite as a Function of the Applied Voltage, 534.
Coil Aërials, Electric Wave Transmission Formulas for Antenna and, *J. H. Dellinger*, 180.
Coils, Inductance, Capacity and Resistance of, at Radio Frequencies, *L. E. Whittemore* and *B. Greit*, 170.
Compton, A. H., A Sensitive Modification of the Quadrant Electrometer: Its Theory and Use, 85.
The Size and Shape of the Electron, 20.
The Size and Shape of the Electron. II. The Absorption of High Frequency Radiation, 247.
Compton, K. T., A Sensitive Modification of the Quadrant Electrometers: Its Theory and Use, 85.
Constants, The Optical, of Liquid Alloys, 459.
Contieri, A. W., Hydrogen Overvoltage, 539.
Crehore, Albert C., Remarks on Dr. L. Silberstein's Results in his Paper "On the Dispersion of the Diamond," 440.
Curtis, H. L., The Use of the Oscillograph for the Study of Ballistic Problems, 183.
Curtiss, L. F., Magneto-Resistance Effects in Films of Bismuth, 536.

D.

Dadourian, H. M., Phonotelemeter; A Device for Locating the Source of a Sound Wave from a Set of Sound-Ranging Data, 178.
Soft X-Rays, 234.
Reflection of Electrons from Metal Surfaces, 434.
Davis, Bergen, Note on a Method of Measuring the Decrease of Velocities in Thin Films by Means of Characteristic X-Rays, 539.
Dawson, Margaret K., The Variation in Sound Intensity of Resonators and Organ Pipes with Blowing Pressure, 49.
Diamond, Remarks on Dr. I. Silberstein's Results in his Paper on the Dispersion of the, *Albert C. Crehore*, 440.
Dellinger, J. H., Electric Wave Transmission Formulas for Antenna and Coil Aërials, 180.
Improvements in Precision Measurements at Radio Frequencies, 181.
Properties of Insulating Materials of the Phenol Type, 199.
Dielectrics, The Reflective Power of Metals and, in the Ultra-violet, *E. P. Lewis* and *A. C. Hardy*, 272.
Diffraction Theory, On the, of Microscopic Vision, *Phanindra Nath Ghosh*, 497.

Displacement from the Apparent Vertical in Free Fall, *Will C. Baker*, 352.
Dorsey, N. E., An Expeditious Graphical Method Applicable to Radium Measurements, 173.
Duane, William, On the Relation between the K Series and the L Series of X-Rays, 67.
On the Critical Absorption and Characteristic Emission X-Ray Frequencies, 369.
On the Spectrum of X-Rays from an Aluminium Target, 389.
On the X-Ray Absorption Frequencies Characteristic of the Chemical Elements, 516.
On the X-Ray Absorption Frequencies Characteristic of the Chemical Elements, 522.
The Relation between the Intensity of General X-Radiation and the Atomic Number of the Anticathode, 525.
Duncan, R. C., The Use of the Oscillograph for the Study of Ballistic Problems, 183.
Duncan, R. W., An Investigation of the Physical Properties of Cotton Airplane Fabrics, 194.

E.

Edlefsen, N. E., Determination of the Normal Temperature by Means of the Equation of the Seasonal-Temperature Variation and of a Modified Thermograph Record, 274.
Electric Furnace, Absorption Effects with the, as Related to Temperature Classification, 270.
Electric Furnace Spectra, The, of Metals in the Infra-Red, 270.
Electric Wave Transmission Formulas for Antenna and Coil Aerials, *J. H. Dellinger*, 180.
Electrical Conduction in Porcelain, *F. B. Silsbee* and *R. K. Honaman*, 197.
Electrical Fume Mask, The, *W. W. Strong*, 176.
Electrical Heating, On the Maintenance of Vibrations of Wires, by, *N. C. Krishmaiyar*, 490.
Electrometer Method, A Note on the Comparison of Inductances, or of an Inductance and a Capacity by an, *Alva W. Smith*, 356.
Electron, The Size and Shape of the, *Arthur H. Compton*, 20.
Electron, The Size and Shape of the, II. The Absorption of High Frequency Radiation, *Arthur H. Compton*, 247.
Electrons, Reflection of, from Metal Surfaces, *H. M. Dadourian*, 434.
Enclosure, Note on the Coefficient of Total Radiation of a Uniformly Heated, *W. W. Coblentz*, 174.
Energy, Note on the Distribution of, by the Visible Spectrum of a Cylindrical Acetylene Flame, *W. W. Coblentz*, 168.
Energy, On the Distribution of, in the Visible Spectrum of an Acetylene Flame, *E. P. Hyde, W. E. Forsythe* and *F. E. Cady*, 379.
Erbium Oxide, The Distribution of Energy in the Spectrum of, *W. S. Mallory*, 54.
Evolution and Absorption of Gases by Glass, *D. Ulrey*, 160.
Ewing, S. P., Determination of the Normal Temperature by Means of the Equation of the Seasonal-Temperature Variation and of a Modified Thermograph Record, 274.
Explosion Waves, A Baroscope for Measuring the Pressure in, *Dayton C. Miller*, 450.

F.

Fairchild, C. O., Pyrometric Practice. I., The Temperature Scale; II., Thermoelectric Pyrometry, 169.
Filters, The Spectral Transmission of, used to Detect Camouflage or Improve Visibility, *K. S. Gibson, E. P. T. Tyndall* and *H. J. McNicholas*, 261.
Flame Propagation, A Study of the Velocity of, in the Cylinders of Aircraft Engines, *D. MacKenzie* and *R. K. Honaman*, 169.
Flares, A Method for the Color Grading of Red, *Irwin G. Priest*, 264.
Fluorescence and Absorption of the Uranyl Acetates, *E. L. Nichols, H. L. Howes* and *Frances G. Wick*, 201.
Fluorescence and Absorption of the Uranyl Sulphates, *E. L. Nichols*, 293.
Foley, Arthur L., The Speed of Sound Pulses in Pipes, 143.
Foote, Paul D., Pyrometric Practice. I., The Temperature Scale; II., Thermoelectric Pyrometry, 169.
Foote, Paul D., Ionization and Resonance Potentials for Electrons in Vapors of Lead and Calcium, 534.
Formulas, Electric Wave Transmission, for Antenna and Coil Aërials, *J. H. Dellinger*, 180.
Fourier's Integral, The Solution of Circuit Problems Mathematical Methods of Investigation Resulting from the Application of, *Thornton C. Fry*, 115.
Forsythe, W. E., On the Distribution of Energy in the Visible Spectrum of an Acetylene Flame, 379.
Free Fall, Displacement from the Apparent Vertical in, *Will C. Baker*, 352.
Fry, Thornton C., The Solution of Circuit Problems Mathematical Methods of Investigation Resulting from the Application of Fourier's Integral, 115.
Furnace Temperature Regulator, A, *Walter P. White* and *Leason H. Adams*, 44.

G.

Gardner, Willard, The Movement of Moisture in Soil by Capillarity, 276.
Gas Viscosity, The End Correction in the Determination of, by the Capillary Tube Method, *A. F. Benton*, 403.

Gases, Evolution and Absorption of, by Glass, *D. Ulrey*, 160.
Gerke, R. H., The Size of Smoke Particles, 172.
Ghosh, Phanindra Nath, On the Diffraction Theory of Microscopic Vision, 497.
Gibson, K. S., The Spectral Composition and Color of Certain High Intensity Searchlight Arcs, 184.
Gibson, K. S., Report on the Applicability of Ultra-Violet Rays to Signaling, 188.
Gibson, K. S., The Spectral Transmission of Filters used to Detect Camouflage or Improve Visibility, 261.
Gorton, W. S., The Use of Subsidiary Spark Gaps in Ignition Systems, 196.
Gray, Arthur W., A Precision Potentiometer, 541.

H.

Hardy, A. C., The Reflective Power of Metals and Dielectrics in the Ultra-violet, 272.
Harrison, T. R., Pyrometric Practice. I., The Temperature Scale; II., Thermo-electric Pyrometry, 169.
Heat, The Measurement of, and the Scope of Carnot's Principle, *Arthur C. Lunn*, 1.
Heat Radiation, The Detection of Invisible Objects by, *Samuel O. Hoffman*, 163.
Heat Treatment, Variations due to, Pre-equilibrium Effects. Studies of the Absorption of Gases by Charcoal, *Harvey B. Lemon*, 281.
Hebb, T. C., The Velocity of Sound and the Ratio of the Specific Heats for Air, 74.
Helium, Resonance and Ionization Potentials for Electrons in the Monatomic Gases, Argon, Neon and, *H. C. Rentschler*, 503.
High Frequency Currents on Wires, *J. O. Mauborgne*, 465.
Hoffman, Samuel O., The Detection of Invisible Objects by Heat Radiation, 163.
Honaman, R. K., A Study of the Velocity of Flame Propagation in the Cylinders of Aircraft Engines, 169.
Honaman, R. K., Electrical Conduction in Porcelain, 197.
Howes, H. L., Fluorescence and Absorption of the Uranly Acetates, 201.
Fluorescence and Absorption of the Uranyl Sulphates, 293.
Hu, Kang-Fuh, On the Critical Absorption and Characteristic Emission X-Ray Frequencies, 369.
Hull, Albert C., The Crystal Structure of Ferro-Magnetic Metals, 539.
Hull, L. M., Applications of the Cathode Ray Tube in Radio Work, 264.
Hyde, E. P., On the Distribution of Energy in the Visible Spectrum of an Acetylene Flame, 379.
Hydrogen Bromide, The Vapor Pressures of Hydrogen Chloride and, Above their Aqueous Solution, *S. J. Bates* and *H. D. Kirschman*, 271.
Hydrogen Chloride, The Vapor Pressures of, Hydrogen Bromide Above their Aqueous Solution, *S. J. Bates* and *H. D. Kirschman*, 271.
Hydrogen Overvoltage, *D. A. MacInnes, Leon Adler* and *A. W. Contieri*, 539.

I.

Incandescent Temperatures, The Reflection Factors of Tungsten at Incandescent Temperatures, *W. Weniger* and *A. H. Pfund*, 427.
Inclinometers, Airplane, *David L. Webster*, 161.
Inductance, Capacity and Resistance of Coils at Radio Frequencies, *L. E. Whittemore* and *G. Breit*, 170.
Inductance, A Note on the Comparison of Inductances, or of an, and a Capacity by an Electrometer Method, *Alva W. Smith*, 356.
Induction, Unipolar, *W. F. G. Swann*, 537.
Ignition Systems, The Use of Subsidiary Spark Gaps in, *W. S. Gorton*, 196.
Infra-red, The Electric Furnace Spectra of Metals in the, *Arthur S. King*, 270.
Infra-Red, Recent Observations of Tube-Arc Spectra, especially in the Infra-red, *Arthur S. King* and *Paul W. Merrill*, 271.
Insulating Materials, Properties of, of the Phenol Type, *J. H. Dellinger* and *J. L. Preston*, 199.
Intensity, The Relation between the, of General X-Radiation and the Atomic Number of the Anticathode, *William Duane* and *Takeo Shimizu*, 525.
Invisible Writing, Optical and Photographic Methods for the Detection of, *Irwin G. Priest* and *E. P. T. Tyndall*, 188.

K.

Kahler, H., The Spectral Photoelectric Sensitivity on Molybdenite as a Function of the Applied Voltage, 534.
Kang-Fuh-Hu, On the X-Ray Absorption Frequencies Characteristic of the Chemical Elements, 516.
Kent, C. V., The Optical Constants of Liquid Alloys, 459.
Keyes, Frederick G., On Establishing of the Absolute Temperature Scale, 542.
King, Arthur S., The Electric Furnace Spectra of Metals in the Infra-red, 270.
Absorption Effects with the Electric Furnace as Related to Temperature Classification, 270.
Recent Observations of Tube-arc Spectra, especially in the Infra-red, 271.
King, R. W., The Calculation of Audion Constants, 532.
Kirschman, H. D., The Vapor Pressures of Hydrogen Chloride and Hydrogen Bromide Above their Aqueous Solution, 271.

Klopsteg, Paul E., A New Chronograph for Measurement of Projectile Velocities and Short Time Intervals, 159.
Kovarik, Alois F., Some Experiments bearing on the Nature of γ-Rays, *Alois F. Kovarik*, 179.
Krishmaiyar, N. C., On the Maintenance of Vibrations of Wires by Electrical Heating, 490
Krypton, The Spectra of, and Xenon in the Infra-Red, *Paul W. Merrill*, 271.

L.

Lemon, Harvey B., Studies of the Absorption of Gases by Charcoal. I. Variations due to Heat Treatment, Preëquilibrium Effects, 281.
Studies of the Adsorption of Gases by Charcoal. II., 394.
Lewis, E. P., The Reflective Power of Metals and Dielectrics in the Ultra-Violet 272.
Loomis, Alfred L., A New Chronograph for Measurement of Projectile Velocities and Short Time Intervals, 159.
Loop Aërial, Development of, for Submarine Radio Communication, *J. A. Willoughby* and *P. D. Lowell*, 193.
Love, Beryl F., The Variation in Sound Intensity of Resonators and Organ Pipes with Blowing Pressure, 49.
Low Temperatures, The Magnetic Properties of Some Rare Earth Oxides at, *E. H. Williams*, 348.
Lowell, P. D., Development of Loop Aërial for Submarine Radio Communication, 193.
L-Radiation, Some Nuclear Charges Calculated for, *Fernando Sanford*, 177.
L-Series, The, in the Tungsten X-Ray Spectrum, *O. B. Overn*, 137.
Lunn, Arthur C., The Measurement of Heat and the Scope of Carnot's Principle, 1.

M.

MacInnes, D. A., Hydrogen Overvoltage, 539.
MacKenzie, D., A Study of the Velocity of Flame Propagation in the Cylinders of Aircraft Engines, 169.
McEwen, George Francis, Physical Constants Pertaining to the Ocean, 272.
McNicholas, H. J., The Spectral Composition and Color of Certain High Intensity Searchlight Arcs, 184.
The Spectral Composition and Color of Certain High Intensity Searchlight Arcs, 184.
The Spectral Transmission of Filters used to Detect Camouflage or Improve Visibility, 261.
Magneto, Notes on the Physics of the High Tension, *F. B. Silsbee*, 196.
Magneto-Resistance Effects in Films of Bismuth, *F. K. Richtmyer*, and *L. F. Curtiss*, 536.
Magnetic Properties, The, of Some Rare Earth Oxides at Low Temperatures, *E. H. Williams*, 348.
Mallory, W. S., The Distribution of Energy in the Spectrum of Erbium Oxide, 54.
Mauborgne, J. O., High Frequency Currents on Wires, 465.
Meggers, W. F., The Spectral Composition and Color of Certain High Intensity Searchlight Arcs, 184.
Merrill, Paul W., A Study of the Relative Intensities of Spectrum Lines in Different Regions of the Arc as Compared with their Behavior in other Sources, 271.
Merrill, Paul W., Recent Observations of Tube-Arc Spectra, Especially in the Infra-red, 271.
Merrill, Paul W., The Spectra of Krypton and Xenon in the Infra-Red, 271.
Metals, The Reflective Power of, and Dielectrics in the Ultra-Violet, *E. P. Lewis* and *A. C. Hardy*, 272.
Metals, The Crystal Structure of Ferro-Magnetic, *Albert W. Hull*, 539.
Metal Surfaces, Reflection of Electrons from, *H. M. Dadourian*, 434.
Microscopic Vision, On the Diffraction Theory of, *Phanindra Nath Ghosh*, 497.
Miller, Dayton C., A Baroscope for Measuring the Pressure in Explosion Waves, 450.
Mohler, F. L., Ionization and Resonance Potentials for Electrons in Vapors of Lead and Calcium, 534.
Moisture, The Movement of, in Soil by Capillarity, *Willard Gardner*, 276.
M-Radiation, Formula for the Wave-Lengths of, *Fernando Sanford*, 275.

N.

Neon, Resonance and Ionization Potentials for Electrons in the Monatomic Gases, Argon, and Helium, *H. C. Rentschler*, 503.
New Books, 470.
Nichols, E. L., Fluorescence and Absorption of the Uranyl Acetates, 201.
Nichols, E. L., Fluorescence and Absorption of the Uranyl Sulphates, 293.
Nitrogen, The Radiating Potentials of, *H. D. Smyth*, 409.
Normal Temperature, Determination of the, by Means of the Equation of the Seasonal Temperature Variation and of a Modified Thermograph Record, 274.
Nuclear Charges, Some, Calculated for L-Radiation, *Fernando Sanford*, 177.

O.

Ocean, Physical Constants Pertaining to the, *George Francis McEwen*, 272.
Optical and Photographic Methods for the Detection of Invisible Writing, *Irwin G. Priest* and *E. P. T. Tyndall*, 188.
Oscillograph, The Use of the, for the Study of Ballistic Problems, *H. L. Curtis* and *R. C. Duncan*, 183.

Overn, O. B., The L-Series in the Tungsten X-Ray Spectrum, 137.

P.

Parachute, The Stability of a Rotating, or Falling Body Rotating about a Vertical Axis of Symmetry, *H. Bateman*, 269.

Pfund, A. H., The Reflection Factors of Tungsten at Incandescent Temperatures, 427.

Phenol Type, Properties of Insulating Materials of the, *J. H. Dellinger* and *J. L. Preston*, 199.

Phonotelemeter; A Device for Locating the Source of a Sound Wave from a Set of Sound-Ranging Data, *H. M. Dadourian*, 178.

Phonotelemeter; Sound Wave, A Device for Locating the Source of a, from a set of Sound-Ranging Data, 178.

Photographic Methods, Optical and, for the Detection of Invisible Writing, *Irwin G. Priest* and *E. P. T. Tyndall*, 188.

Photography, The Application of Motion-Picture, to Ballistics and Airplane Performances, *F. C. Brown*, 531.

Photometer, A Precision, *Arthur W. Gray*, 541.

Physical Constants Pertaining to the Ocean, *George Francis McEwen*, 272.

Pilling, Norman B., The Quenching Behavior of Liquids, 222.

Pipes, The Speed of Sound Pulses in, *Arthur L. Foley*, 143.

Planck, Constant H. The Most Probable Value of the, *Raymond T. Birge*, 361.

Porcelain, Electrical Conduction in, *F. B. Silsbee* and *R. K. Honaman*, 197.

Portable Volume Standard for Gases, *M. H. Stillman*, 174.

Potentials, Ionization and Resonance, for Electrons in Vapors of Lead and Calcium, *E. L. Mohler, Paul D. Foote*, and *H. F. Stimson*, 534.

Potentials, Resonance and Ionization, for Electrons in the Monatomic Gases, Argon, Neon and Helium, *H. C. Rentschler*, 503.

Precision Measurements, Improvements in, at Radio Frequencies, *J. H. Dellinger*, 181.

Preston, J. L., Properties of Insulating Materials of the Phenol Type, 199.

Priest, Irwin G., A Method for the Color Grading of Red Flares, 264.

Priest, Irwin G., A New Formula for the Spectral Distribution of Energy from a Complete Radiator. II., Concerning the Value of the Constant D_2, 191.

Priest, Irwin G., Optical and Photographic Methods for the Detection of Invisible Writing, 188.

Priest, Irwin G., Report of the Applicability of Ultra-violet Rays to Signaling, 188.

Priest, Irwin G., The Spectral Composition and Color of Certain High Intensity Searchlight Arcs, 184.

Projectile Velocities, A New Chronograph for Measurement of, and Short Time Intervals, 159.

Pyrometric Practice. I., The Temperature Scale; II., Thermoelectric Pyrometry. *Paul D. Foote, T. R. Harrison* and *C. O. Fairchild*, 169.

Pyrometry, Thermoelectric, Pyrometric Practice. I., The Temperature Scale, *Paul D. Foote, T. R. Harrison* and *C. O. Fairchild*, 169.

Q.

Quadrant Electrometer, A Sensitive Modification of the, Its Theory and Use, *A. H. Compton* and *K. T. Compton*, 85.

Quenching Behavior of Liquids, *Norman B. Pilling*, 222.

R.

Radiating Potentials, The, of Nitrogen, *H. D. Smyth*, 409.

Radiation Formulæ, Comments on Spectral, 267.

Radio Communication, Development of Loop Aërial for Submarine, *J. A. Willoughby* and *P. D. Lowell*, 193.

Radio Frequencies, Inductance, Capacity and Resistance of Coils at, *L. E. Whittemore* and *G. Breit*, 170.

Radio Frequencies, Improvements in Precision Measurements at, *J. H. Dellinger*, 181.

Radium Measurements, An Expeditious Graphical Method Applicable to, *N. E. Dorsey*, 173.

Radio Work, Applications of the Cathode Ray Tube in, *L. E. Whittemore* and *L. M. Hull*, 266.

Raman, C. V., An Experimental Method for the Production of Vibrations, 446.

Reflection Factors, The, of Tungsten at Incandescent Temperatures, *W. Weniger* and *A. H. Pfund*, 427.

Reflection of Electrons from Metal Surfaces, *H. M. Dadourian*, 434.

Reflective Power, The, of Metals and Dielectrics in the Ultra-Violet, *E. P. Lewis* and *A. C. Hardy*, 272.

Relative Intensities, A Study of the, of Spectrum Lines in Different Regions of the Arc as Compared with their Behavior in other Sources, *Paul W. Merrill*, 271.

Rentschler, H. C., Resonance and Ionization Potentials for Electrons in the Monatomic Gases Argon, Neon and Helium, 503.

Resistance, Inductance, Capacity and, of Coils at Radio Frequencies, *L. E. Whittemore* and *G. Breit*, 170.

Richards, Horace C., The Motion of a Rotating Projectile, 198.

Richtmyer, F. K., Magneto-Resistance Effects in Films of Bismuth, 536.

Rotating Projectile, The Motion of a, *Horace C. Richards*, 198.

S.

Sanford, Fernando, Formula for the Wave-lengths of *M*-Radiation, 275.
Some Nuclear Charges Calculated for L-Radiation, 177.
Searchlight Arcs, The Spectral Composition and Color of Certain High Intensity, by *Irwin G. Priest, W. F. Meggers, H. J. McNicholas, K. S. Gibson* and *E. P. T. Tyndall* in coöperation with the Searchlight Investigation Section, Corps of Engineers, U. S. A., 184.
Searchlight Investigation Section, The Spectral Composition and Color of Certain High Intensity Searchlight Arcs, 184.
Shimizu, Takeo, On the Relation between the K Series and the L Series of X-Rays, 67.
Shimizu, Takeo, On the Spectrum of X-Rays from an Aluminium Target, 389.
Shimizu, Takeo, On the X-Ray Absorption Frequencies Characteristic of the Chemical Elements, 522.
Shimizu, Takeo, The Relation between the Intensity of General X-Radiation and the Atomic Number of the Anticathode, 525.
Signaling, Report on the Applicability of Ultra-Violet Rays to, *Irwin G. Priest* and *K. S. Gibson*, 188.
Silberstein's Results, Remarks on Dr. L., in his Paper "On the Dispersion of the Diamond," *Albert C. Crehore*, 440.
Silsbee, F. B., Electrical Conduction in Porcelain, 197.
Notes on the Physics of the High Tension Magneto, 196.
Slaughter, N. H., The Production of Vacuum Tubes for Military Purposes, 467.
Smith, Alva W., A Note on the Comparison of Inductances, or of an Inductance and a Capacity by an Electrometer Method, 356.
Smoke Particles, The Size of, *P. V. Wells* and *R. H. Gerke*, 172.
Smyth, H. D., The Radiating Potentials of Nitrogen, 409.
Solids, Temperature Distribution in, during Heating or Cooling, *E. D. Williamson* and *L. H. Adams*, 99.
Sound Intensity, The Variation in, of Resonators and Organ Pipes with Blowing Pressure, *Beryl F. Love* and *Margaret K. Dawson*, 49.
Sound, Location of Aircraft by, *G. W. Stewart*, 166.
Sound, Propagation of, in an Irregular Atmosphere, *G. W. Stewart*, 376.
Sound Pulses, The Speed of, in Pipes, *Arthur L. Foley*, 143.
Spark Gaps, The Use of Subsidiary, in Ignition Systems, *W. S. Gorton*, 196.
Specific Heats, The Velocity of Sound and the Ratio of the, for Air, *T. C. Hebb*, 74.
Spectral Distribution, A New Formula for the, of Energy from a Complete Radiator. II., Concerning the Value of the Constant D_2, *Irwin G. Priest*, 191.
Spectral Transmission, The, of Filters Used to Detect Camouflage or Improve Visibility, *K. S. Gibson, E. P. T. Tyndall* and *H. J. McNicholas*, 261.
Spectrum Lines, A Study of the Relative Intensities of, in Different Regions of the Arc as Compared with their Behavior in other Sources, *Paul W. Merrill*, 271.
Spectrum, On the Distribution of Energy in the Visible, of an Acetylene Flame, *F. E. Cady*, 379.
Spectrum, The Distribution of Energy in the, of Erbium Oxide, *W. S. Mallory*, 54.
Speed, The, of Sound Pulses in Pipes, *Arthur L. Foley*, 143.
Stability, The, of a Rotating Parachute or Falling Body Rotating about a Vertical Axis of Symmetry, *H. Bateman*, 269.
Stark Effect, The, for Metals in the Ultra-Violet, *J. A. Anderson*, 270.
Stewart, G. W., Location of Aircraft by Sound, 166.
Stewart, G. W., Propagation of Sound in an Irregular Atmosphere, 376.
Stillman, M. H., Photographic Method of Detecting Changes in a Complicated Group of Objects with Especial Reference to Camouflage Detection, 173.
Portable Volume Standard for Gases, 174.
Stimson, H. F., Ionization and Resonance Potentials for Electrons in Vapors of Lead and Calcium, 534.
Strong, W. W., The Electrical Fume Mask, 176.
Swann, W. F. G., Unipolar Induction, 537.

T.

Temperature Distribution in Solids During Heating or Cooling, *E. D. Williamson* and *L. H. Adams*, 99.
Temperature Regulator, A Furnace, *Walter P. White* and *Leason H. Adams*, 44.
Temperature Scale, I The; II Thermoelectric Pyrometry. Pyrometric Practice, *Paul D. Foote, T. R. Harrison* and *C. O. Fairchild*, 169.
Thermodynamic Discussion, A Critical, of the Volta, Thermo-electric and Thermionic Effects, *P. W. Bridgman*, 306.
Thermionic Effects, A Critical Thermodynamic Discussion of the Volta, Thermo-eleccric and, *P. W. Bridgman*, 306.
Thermo-electric, A Critical Thermodynamic Discussion of the Volta, and Thermionic Effects, *P. W. Bridgman*, 306.
Time Intervals, A New Chronograph for Measurement of Projectile Velocities and Short, *Paul E. Klopsteg* and *Alfred L. Loomis*, 159.
Total Radiation, Note on the Coefficient of of a Uniformly Heated Enclosure, *W. W. Coblentz*, 174.

Tube-Arc Spectra, Recent Observations of, especially in the Infra-red, *Arthur S. King* and *Paul W. Merritt*, 271.
Tungsten X-Ray Spectrum, The L-Series in the, *O. B. Overn*, 137.
Tungsten, The Reflection Factors of, at Incandescent Temperatures, *W. Weniger* and *A. H. Pfund*, 427.
Tyndall, E. P. T., The Spectral Composition and Color of Certain High Intensity Searchlight Arcs, 184.
Tyndall, E. P. T., Optical and Photographic Methods for the Detection of Invisible Writing, 188.
Tyndall, E. P. T., The Spectral Transmission of Filters Used to Detect Camouflage or Improve Visibility, 261.

U.

Ulrey, D., Evolution and Absorption of Gases by Glass, 160.
Ultra-Violet Rays, Report on the Applicability of Ultra-Violet Rays to Signaling, *Irwin G. Priest* and *K. S. Gibson*, 188.
Ultra-Violet, The Stark Effect for Metals in the, *J. A. Anderson*, 270.
Uranyl Acetates, Fluorescence and Absorption of the, *E. L. Nichols, H. L. Howes* and *Frances G. Wick*, 201.
Uranyl Sulphates, Fluorescence and Absorption of the, *E. L. Nichols* and *H. L. Howes*, 293.

V.

Vacuum Tube, The Production of, for Military Purposes, *N. H. Slaughter*, 467.
Veblen, O., Progress in Design of Artillery Projectiles, 167.
Velocity of Sound, The, and the Ratio of the Specific Heats for Air, *T. C. Hebb*, 74.
Velocity, A Study of the, of Flame Propagation in the Cylinders of Aircraft Engines, *D. MacKenzie* and *R. K. Honaman*, 169.
Vibrations, An Experimental Method for the Production of, *C. V. Raman*, 446.
Vibrations of Wires, On the Maintenance of, by Electrical Heating, *N. C. Krishmaiyar*, 490.
Visible Spectrum, Note on the Distribution of Energy by the, of a Cylindrical Acetylene Flame, *W. W. Coblentz*, 168.
Visibility, The Spectral Transmission of Filters Used to Detect Camouflage or Improve, *K. S. Gibson, E. P. Tyndall* and *H. J. McNicholas*, 261.
Volta, A Critical Thermodynamic Discussion of the, Thermo-electric and Thermionic Effects, *P. W. Bridgman*, 306.

W.

Webster, David L., Airplane Inclinometers, 161.
Wells, P. V., The Size of Smoke Particles, 172.
Weniger, W., The Reflection Factors of Tungsten at Incandescent Temperatures, 427.
West, Frank L., Determination of the Normal Temperature by Means of the Equation of the Seasonal-Temperature Variation and of a Modified Thermograph Record, 274.
White, Walter P., A Furnace Temperature Regulator, 44.
Whittemore, L. E., Inductance, Capacity and Resistance of Coils at Radio Frequencies, 170.
Whittemore, L. E., Applications of the Cathode-Ray Tube in Radio Work, 266.
Wick, Frances G., Fluorescence and Absorption of the Uranyl Acetates, 201.
Williams, E. H., The Magnetic Properties of Some Rare Earth Oxides at Low Temperatures, 348.
Williamson, E. D., Temperature Distribution in Solids during Heating or Cooling, 99.
Willoughby, J. A., Development of Loop Aërial for Submarine Radio Communication, 193.
Wires, High Frequency Currents on, 465.

X.

Xenon, The Spectra of Krypton and, in the Infra-Red, *Paul W. Merrill*, 271.
X-Rays, Soft, *H. M. Dadourian*, 234.
X-Rays, On the Spectrum of, from an Aluminium Target, *William Duane* and *Takeo Shimizu*, 389.
X-Rays, Note on a Method of Measuring the Decrease of Velocities in Thin Films by Means of Characteristic, *Bergen Davis*, 539.
X-Ray Absorption Frequencies, On the, of the Chemical Elements, *William Duane* and *Kang-Fuh-Hu*, 516.
X-Ray Absorption Frequencies, On the, Characteristic of the Chemical Elements, *William Duane* and *Takeo Shimizu*, 522.
X-Ray Frequencies, On the Critical Absorption and Characteristic Emission, *William Duane* and *Kang-Fuh Hu*, 369.
X-Radiation, The Relation between the Intensity of General, and the Atomic Number of the Anticathode, *William Duane* and *Takeo Shimizu*, 525.
X-Ray Spectrum, The L-Series in the Tungsten, *O. B. Overn*, 137.
γ-Rays, Some Experiments Bearing on the Nature of, *Alois F. Kovarik*, 179.

LEEDS & NORTHRUP

CAPACITY BRIDGE

FOR RAPID AND ACCURATE MEASUREMENT OF CONDENSER CAPACITIES

SIMPLE TO OPERATE:

1. Connect the source of high frequency alternating current, the telephone receiver and the unknown capacity, to designated binding posts.

2. Turn the knob until minimum tone is heard in the receiver.

3. Read the scale and multiply reading by the standard capacity value.

BEST RANGE .001 TO 10 MICROFARADS:

Two self-contained mica standard condensers of .01 and .1 microfarad, respectively, enable accurate determinations tc be made within the range mentioned. A pair of extra binding posts permits of the use of an external standard capacity of value below or above that range.

The cost of the bridge with a protecting cover, not shown in the cut, is $65.00.

Bulletin P120 tells more about it

THE LEEDS & NORTHRUP CO.

ELECTRICAL MEASURING INSTRUMENTS

4901 STENTON AVENUE PHILADELPHIA, PA.

THE PHYSICAL REVIEW

Information for Contributors

1. The purpose of the REVIEW is to publish articles that add to our knowledge of experimental or theoretical physics. Such articles may be submitted by American and other physicists, whether members of THE AMERICAN PHYSICAL SOCIETY or not, but no article will be considered unless the manuscript is in English and in a form ready for publication.

All correspondence relating to contributions should be addressed to THE PHYSICAL REVIEW, Ithaca, N. Y. Manuscripts will be acknowledged as soon as received. Articles submitted for publication in the REVIEW are not to be submitted for publication elsewhere unless the authorization of the editors has been obtained, and, in case of duplicate publication in English, proper reference to the REVIEW is made. Manuscript should be ready for the printer; the editors cannot assume responsibility for its correctness. The presentation should be as concise as is consistent with clearness; all unnecessary duplication of data in table and in curves is to be avoided, and tabular matter should be introduced *in extenso* only when the exact numerical values affect the value of the paper. It is requested that the metric system, be used in all cases. Whenever the value of a quantity is expressed in any other system, its value in the metric system should be added in parentheses. *Every article is to be preceded by a synopsis,* information concerning which may be obtained from the editor.

To insure prompt attention during the summer, manuscripts should be submitted by June 15.

2. Illustations should be in black and white and should be ready for direct reproduction; such illustrations will be made without expense to the author. The ink used in preparing illustrations should be *jet black.* Curves can be satisfactorily reproduced when plotted on plain paper or on blue-lined-section paper; co-ordinates may be ruled in black at desired intervals, for example every centimeter or every inch. Blue lines are not reproduced photographically; *colors other than jet-black and blue should be avoided.* The material in an illustration should be compactly arranged; when it is much spread out, a greater reduction is necessary in reproduction. Lettering on illustrations should be plain and *of sufficient size to be legible* after reduction and should be complete (including designation of co-ordinates), requiring no printing from type except in the descriptive caption beneath.

It is generally desirable to refer to *all* illustrations as "figures," designated by some one consecutive series of numbers. The location of each figure should be marked in the manuscript, together with any caption which is to be printed beneath it.

3. A proof of each contributed article will be sent to the author, if in America, for his approval. This is to afford opportunity for making corrections and not for extensive changes; such changes, if made, may be charged to the author. It is requested that all proof be returned promptly. When proof is not so returned, after thirty days an article may be printed without proof-reading by the author.

Authors should note that cross reference can not well be made by page number, for a change in paging is often necessary in the final make-up.

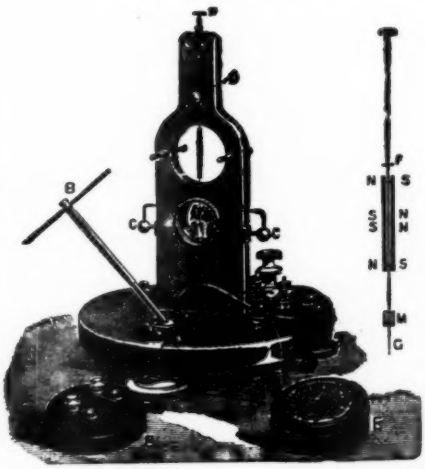
Although proof of abstracts can usually be submitted to authors, this is not always possible without delay in publication; in the case of abstracts, therefore, it is particularly important that the manuscript be free from error. In revising proof of abstracts, authors should correct any errors of the printer, but should make no changes that will affect the arrangement of paging.

4. Offprints, ordered on the proper form with return of proof, will be furnished by the printer according to the prices given below. Any special instructions in regard to offprints—special title page, etc.,—should be indicated on the order to the printer (or attached thereto) and not as a letter to the editors. (Further inquiries in regard to offprints may be addressed to The New Era Printing Co., Lancaster, Pa.)

PRICE LIST FOR REPRINTS

	4pp.	8pp.	12pp.	16pp.	20pp.	24pp.	28pp.	32pp.	48pp.	64pp.
25 copies.....	$1.40	$2.46	$3.65	$4.40	$5.65	$6.50	$8.00	$8.45	$12.55	$15.90
50 "	1.65	2.90	4.25	5.10	6.65	7.75	9.40	9.35	14.15	17.35
75 "	1.95	3.35	4.85	5.65	7.60	8.75	10.45	11.25	15.65	19.95
100 "	2.25	3.80	5.35	6.35	8.25	9.80	11.55	12.45	17.55	22.05
150 "	2.70	4.60	6.50	7.60	10.20	12.10	14.20	15.20	21.35	26.80
200 "	3.50	5.05	7.15	8.35	11.40	13.50	15.80	16.85	23.55	29.60
300 "	3.85	6.20	9.20	10.70	14.85	17.55	20.50	21.05	30.20	37.40

The Broca Galvanometer



A Sensitive, Astatic Moving-Magnet Galvanometer

Interchangeable coils of different resistances, which can be connected in series or in parallel, provide a ready means of varying the resistance of the instrument.

The period and damping can also be easily altered.

Full particulars, sensitivity data, etc., are given in List No. 126A.

CAMBRIDGE SCIENTIFIC INSTRUMENT CO., Ltd.
CAMBRIDGE, ENGLAND

GRAPHITE-SELENIUM CELLS

Fournier d'Albe's Pattern

Great Stability and High Efficiency

With a sensitive selenium surface of 5 sq. cm., and a voltage of 20, the additional current obtainable at various illuminations (in metre-candles) is:

At 1 m.c. . . .	¼	milliamp
At 50 m.c. . . .	1	"
At 500 m.c. . .	2	"

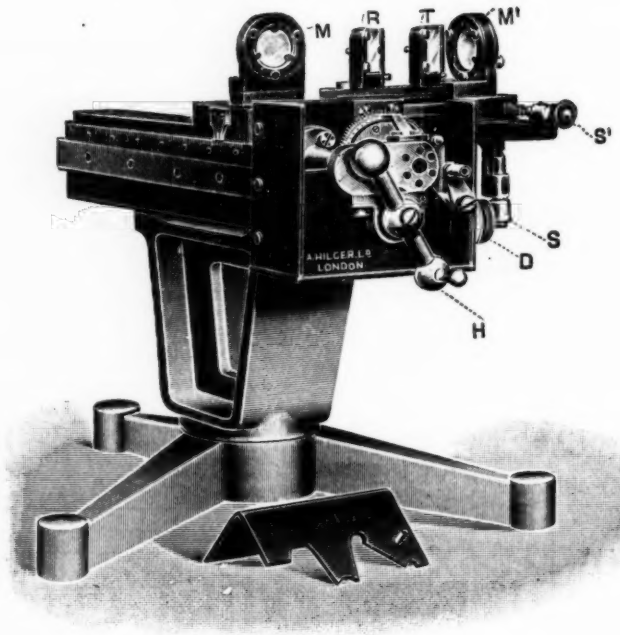
For particulars and prices apply to the SOLE AGENTS

JOHN J. GRIFFIN & SONS LTD

Makers of Physical, Electrical & Scientific Instruments

KEMBLE ST. KINGSWAY LONDON, W. C.

Michelson and Fabry & Perot Interferometers



Our latest models of **Michelson** and **Fabry & Perot Interferometers** (see figure) are in every case so arranged that either the Fabry & Perot or the Michelson system of mirrors can be supplied at any subsequent time and attached without any alteration of the apparatus.

Three sizes are made, having respectively 75 mm., 120 mm., and 200 mm. travel of the movable mirror, and a set of mirrors suitable for any one size is also suitable for any other.

Full particulars post free on application to

ADAM HILGER, Ltd.

75 A. Camden Road **LONDON, N.W.**

Telegraphic address "Sphericity, London." Cable Code---Western Union.

THE CUTLER-HAMMER MFG. CO.

ELECTRIC CONTROLLING DEVICES

MILWAUKEE, WIS.

VICTOR ELECTRIC CORPORATION

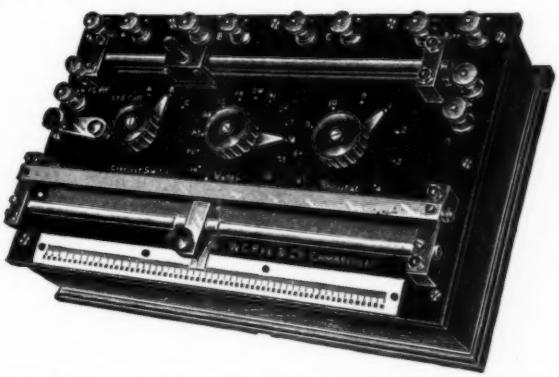
Manufacturers of

X-Ray and Other High Potential Apparatus

Main Office and Factory:

236 So. Robey Street CHICAGO

"Granta Minor" Potentiometer



A HIGH GRADE Instrument at a VERY MODERATE PRICE

Made in Two Ranges: .0002 Volt to 1.7 Volts and 5 Micro-volts to 34 Milli-volts

Price, $45.60 and $48.00 respectively

Write for particulars and discount.

W. G. PYE & COMPANY

Scientific Instrument Makers

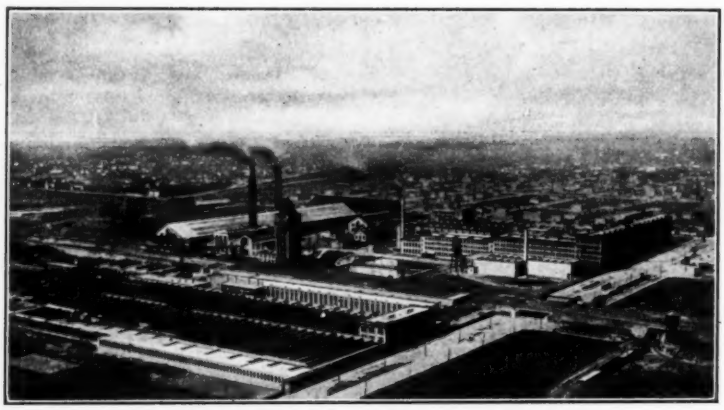
"Granta" Works Montague Road, Cambridge, Eng.

(3)

Opportunity

with the

Western Electric Company
INCORPORATED



The Hawthorne (Illinois) Works where over 20,000 people are employed

In every branch of the Western Electric Company's activities there is opportunity for the trained man — physicist, engineer, chemist, designer, draftsman—opportunity for development and advancement

The Company is engaged in research, development and design problems affecting the telephone, telegraph and radio and manufactures the apparatus used in these systems of communication.

In addition to apparatus of its own manufacture, the Company distributes a complete line of electrical supplies. For this purpose forty branch houses are maintained throughout the United States.

Write for our booklet "Opportunity" and learn more in detail what the Company offers the trained man.

Western Electric Company
INCORPORATED

195 Broadway, New York, N. Y.

And other principal cities

A Bit Unusual

The paper you are using now may be different from our standard cross section papers. Perhaps our stock papers will do and you can save money. If our plate will, but the size of paper is different, that can be overcome by ordering in lots of five hundred sheets or over. Have you our sample book?

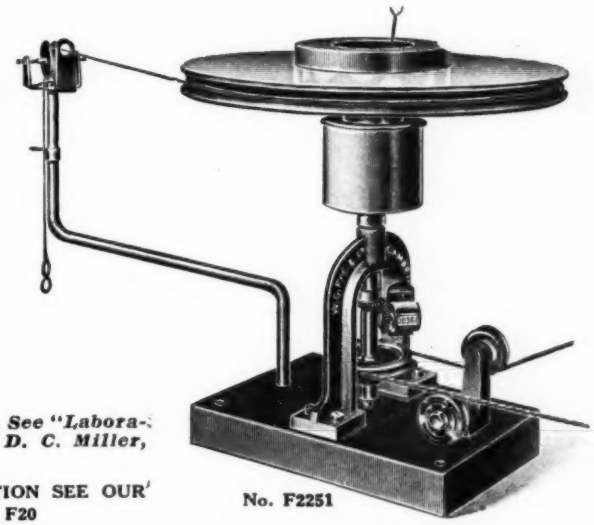
CORNELL CO-OPERATIVE SOCIETY

MORRILL HALL ***ITHACA, N. Y.***

THE MECHANICAL EQUIVALENT OF HEAT

is Readily Determined by College Students with a High Degree of Accuracy by Means of Puluj's Method.

The Apparatus, as Shown Here, is Carefully Made in Our Own Instrument Shop and Nicely Finished.



For Method of Use, See "Laboratory Physics" by D. C. Miller, page 194.

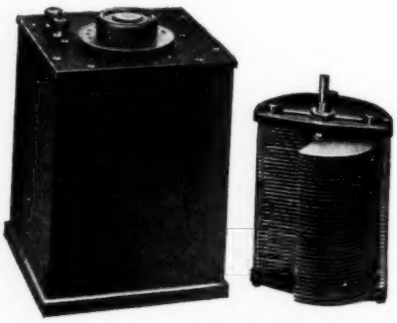
FOR FULL DESCRIPTION SEE OUR CATALOG F20

No. F2251

CENTRAL SCIENTIFIC COMPANY

460 East Ohio Street

CHICAGO **U. S. A.**



VARIABLE AIR CONDENSER TYPE 101

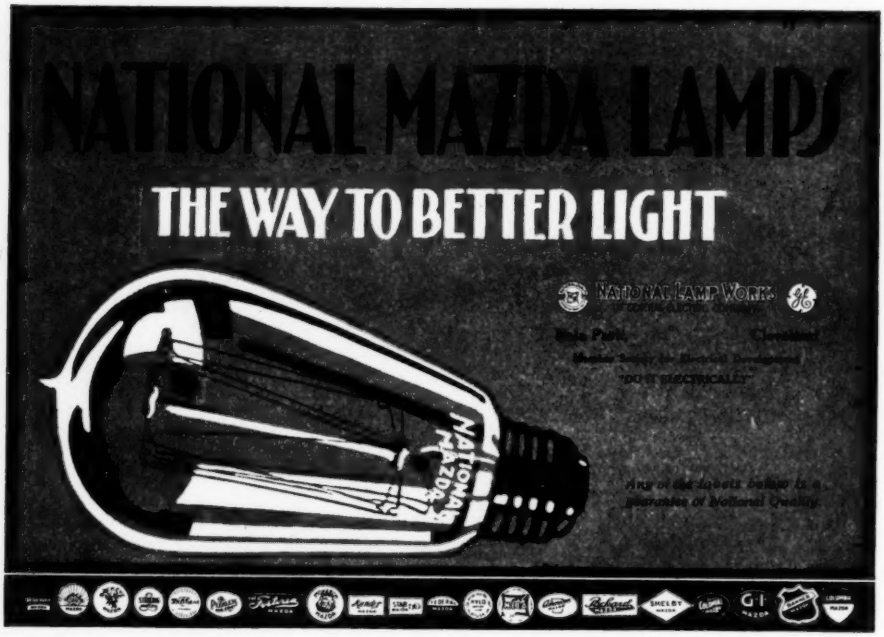
This Condenser is representative of the HIGH GRADE
LINE OF

Radio Laboratory Apparatus

MANUFACTURED BY

GENERAL RADIO CO.

11 WINDSOR STREET CAMBRIDGE, MASS.



MORSE TWIST DRILL & MACHINE COMPANY New Bedford, Mass.

Makers of Twist Drills, Reamers, Cutters, Etc.

TOOLS THAT PROVE THEIR WORTH.

Laboratory Spectrometers

SPECIALTIES

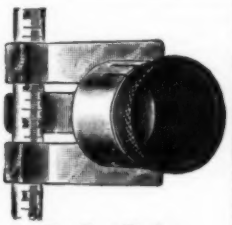
High Grade Measuring Instruments to cover all requirements

Laboratory and demonstration apparatus for advanced and elementary work

Full line Calorimeters

Universal Laboratory Supports

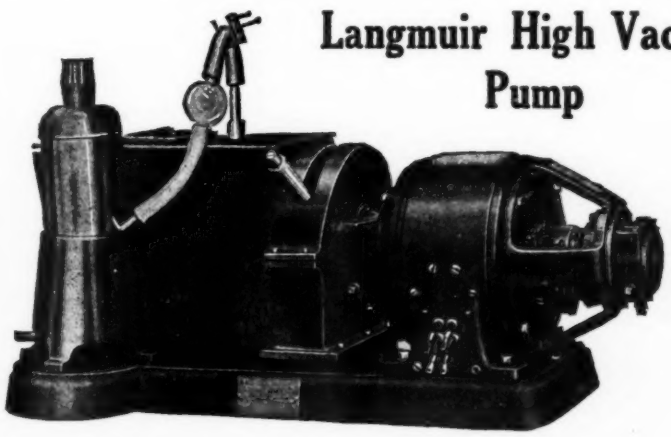
General Laboratory Supplies



Reading Device for Thermometers $3.00

WM. GAERTNER & CO., 5345-49 Lake Park Ave., Chicago

Langmuir High Vacuum Pump



We offer **high-speed high-vacuum** pumps, with auxiliary oil-sealed pump, and D.C. or A.C. motor—all mounted together on one bed plate, as illustrated above; or alone, without accessory equipment.

Write for Descriptive Bulletin 879

JAMES G. BIDDLE

1211 Arch Street PHILADELPHIA

RESEARCH LABORATORY
SCHENECTADY, N. Y.

The Service of an Electrical Research Laboratory

In the research laboratories of the General Electric Company are many scientists working to increase the usefulness of electricity to industry.

In one of these laboratories at Schenectady are nearly 70,000 sq. ft. of floor space, with each room piped for water, gas, compressed air, vacuum, high and low pressure hydrogen, oxygen and high pressure steam, and adjoining are completely manned and equipped machine and forge shops, etc.

In this laboratory research work has been done on paints, oils and varnishes; irons, steels and alloys; copper, zinc, molybdenum and magnesium; kenotrons and pliotrons for radio work; X-ray location of holes in castings; incandescent lamps and searchlights; insulation and brush compounds; turbine blades and boiler feed water; atomic hydrogen; molecular layers in catalyzers; fuse fillers; boron, uranium and thorium; rubber and platinum substitutes, sherardizing and calorizing, condenser, boiler and pyrometer tubes and powerful X-ray tubes.

It is the sincere desire of this Company to co-operate to the fullest extent with engineers toward the solution of any electrical problem.

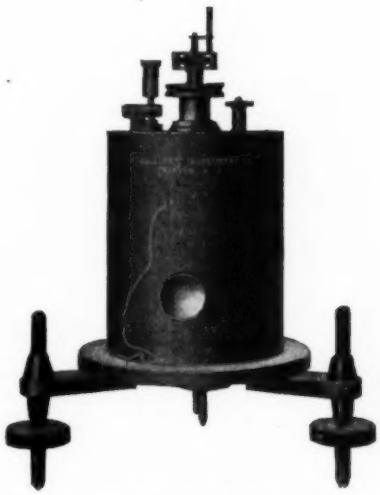
General Electric Company

General Office
Schenectady, N.Y.

Sales Offices in all large cities

95-148

COMPTON QUADRANT ELECTROMETER



Compton Quadrant Electrometer Catalog No. 450

SENSITIVITY:

The total range of the instrument is approximately 0–50,000 mm. per volt, and the easy working range is 0–20,000 mm. per volt. Theoretically, infinite sensitivity with infinite period could be reached with infinite skill in adjustment. Deflections are proportional to the voltages over a very wide range.

CAPACITY:

Because of the relatively small size of the electrical system, the instrument has the advantage of a very small capacity (about 12 cm.).

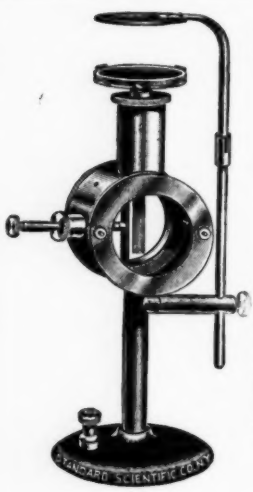
Described and Listed in Circular No. 16. Write for it

PYROLECTRIC INSTRUMENT CO.

ELECTRICAL PRECISION INSTRUMENTS

636-640 East State Street **TRENTON, N. J.**

E. F. Northrup, President and Technical Adviser



Stansico-Zeleny Oscillating Electroscope

For Radioactivity and other Ionization Experiments

Qualitative and Quantitative

Utilizes the novel principle of measuring the period of pendular vibration of the leaf instead of the old method of the rate of motion of gold leaves. Wide range of application, great sensitivity and accuracy.

Send for descriptive circular P1

STANDARD SCIENTIFIC COMPANY

70 Fifth Avenue **NEW YORK, N. Y.**

Scientific Instruments and Laboratory Supplies

PAST OFFICERS
OF
THE AMERICAN PHYSICAL SOCIETY

PRESIDENT

Henry A. Rowland	1899–1900	W. F. Magie	1911–1912
Albert A. Michelson	1901–1902	B. O. Peirce	1913
Arthur G. Webster	1903–1904	Ernest Merritt	1914–1915
Carl Barus	1905–1906	R. A. Millikan	1916–1917
Edward L. Nichols	1907–1908	H. A. Bumstead	1918
Henry Crew	1909–1910	J. S. Ames	1919–

VICE-PRESIDENT

Albert A. Michelson	1899–1900	W. F. Magie	1909–1910
Arthur G. Webster	1901–1902	B. O. Peirce	1911–1912
Elihu Thomson	1903–1904	Ernest Merritt	1913
D. B. Brace	1905	K. E. Guthe	1914–1915
E. L. Nichols	1906	H. A. Bumstead	1916–1917
Henry Crew	1907–1908	J. S. Ames	1918
W. C. Sabine, T. Lyman	1919		

SECRETARY

Ernest Merritt	1899–1912	A. D. Cole	1913–1917
Dayton C. Miller	1918–		

TREASURER

William Hallock	1899–1907	J. S. Ames	1907–1917
G. B. Pegram	1918–		

ELECTED MEMBERS OF THE COUNCIL

H. S. Carhart, M. I. Pupin	1899–1900	J. C. McLennan, E. F. Nichols	1909–1912
W. F. Magie	1899–1901, 1902–1906	K. E. Guthe, C. A. Skinner	1910–1913
B. O. Peirce	1903–1906	R. A. Millikan, John Zeleny	1911–1914
A. G. Webster	1899–1901, 1905–1908	H. A. Bumstead, Theodore Lyman	1912–1915
C. S. Hastings	1899–1902	H. T. Barnes, W. J. Humphreys	1913–1916
A. L. Kimball	1901–1902	G. K. Burgess, D. C. Miller	1914–1917
J. S. Ames	1899–1903, 1905–1908	C. E. Mendenhall, G. W. Stewart	1915–1918
E. L. Nichols	1899–1903	Irving Langmuir, Geo. B. Pegram	1916–1919
Henry Crew, E. B. Rosa	1901–1904	G. O. Squier, H. A. Wilson	1917–1920
Carl Barus, D. B. Brace	1902–1904	A. L. Day, G. F. Hull	1918–1921
E. P. Lewis, Francis E. Nipher	1905	Max Mason	1919
E. H. Hall	1903–1906	G. K. Burgess, J. C. McLennan	1919–1922
W. S. Franklin, Ernest Rutherford	1904–1907		
R. A. Millikan, A. Trowbridge	1906–1907		
H. A. Bumstead, C. E. Mendenhall	1907–1910		
W. C. Sabine, S. W. Stratton	1908–1911		

MANAGING EDITOR

Frederick Bedell 1913–

BOARD OF EDITORS

J. S. Ames, B. O. Peirce, John Zeleny	1913	J. S. Ames, Edgar Buckingham, A. A. Michelson	1915–1917
K. E. Guthe, W. F. Magie, C. A. Skinner	1913–1914	E. P. Lewis, W. C. Sabine, A. Trowbridge	1916–1918
J. C. McLennan, R. A. Millikan, E. F. Nichols	1913–1915	N. E. Dorsey, Wm. Duane, O. M. Stewart	1917–1919
H. A. Bumstead, C. E. Mendenhall, A. G. Webster	1914–1916	G. K. Burgess, A. D. Cole, A. C. Lunn	1918–1920
Henry Crew, L. V. King, H. S. Uhler	1919–1921		

(CONTENTS—Continued)

Proceedings of the American Physical Society. 530

Minutes of the Ninety-Ninth Meeting; The Application of Motion-Picture Photography to Ballistics and Airplane Performance, *F. C. Brown*; The Calculation of Audion Constants, *R. W. King*; Ionization and Resonance Potentials for Electrons in Vapors of Lead and Calcium, *F. L. Mohler, Paul D. Foote and H. F. Stimson*; The Spectral Photoelectric Sensitivity on Molybdenite as a Function of the Applied Voltage, *W. W. Coblentz and H. Kahler*; Magneto-Resistance Effects in Films of Bismuth, *F. K. Richtmyer and L. F. Curtiss*; Unipolar Induction, *W. F. G. Swann*; Note on a Method of Measuring the Decrease of Velocities in Thin Films by Means of Characteristic X-Rays, *Bergen Davis*; Hydrogen Overvoltage, *D. A. MacInnes, Leon Adler and A. W. Contieri*; The Crystal Structure of Ferro-Magnetic Metals, *Albert W. Hull*; On Establishing of the Absolute Temperature Scale, *Frederick G. Keyes*; A Precision Potentiometer, *Arthur W. Gray*.